

Fourth-Year Capstone Courses

The American Diploma Project (ADP) research has found students must master a common core of knowledge and skills in order to be prepared for success in postsecondary education and the world of work. In mathematics, this content—as represented in the ADP Benchmarks—reflects expectations typically found in a rigorous course sequence that includes courses such as Algebra I, Algebra II, and Geometry, or their equivalents, as well as considerable data analysis and statistics. While some students will require all four years of high school to complete this rigorous course of study, many will complete this work in their second or third year. The National Council of Teachers of Mathematics (NCTM), Achieve, ACT, the College Board, and others recommend that all students study mathematics in each of the four years they are enrolled in high school. Research has shown that a fourth year of mathematics study in high school is positively associated with higher scores on the ACT and SAT exams, as well as lower postsecondary remediation rates.

For students intending to pursue postsecondary training or careers in the science, technology, engineering, and mathematics (STEM) fields, some excellent advanced fourth-year mathematics courses already exist. These include ACT’s rigorous precalculus course, for which ACT has recently published a model course syllabus, Advanced Placement Calculus, and International Baccalaureate’s Higher Level Mathematics. These courses are often well-aligned with the rigorous mathematics courses students will encounter upon entering college.

There remain, however, a significant number of high school students who may be more interested in a contextualized form of instruction in their senior year or are not yet ready for the demands of a traditional precalculus or calculus course. In the past, many of these students have opted out of mathematics beyond Algebra II or its equivalent. These students would benefit greatly from an alternative capstone experience—one that maintains and extends prior mathematical knowledge, enhances the application of process skills, encourages the development of academic discipline and a positive attitude toward learning mathematics, and connects mathematics with varied student interests. These courses will keep students engaged in learning and ensure a seamless transition to further education and the workplace.

It is the goal of this project to provide policymakers and educators with criteria that they may use to develop and evaluate alternative capstone course curricula. We have also provided links to and descriptions of a number of fourth-year capstone courses that are currently in use or under development. We hope that, in time, additional capstone courses will be identified, evaluated, and shared through this website.

Criteria for High-Quality Capstone Courses

The following rating system will be used to judge the quality of a course curriculum:

0

= **There is no basis in the course description for evaluation of this criterion.**

1

= **Criterion is not suggested overtly but is implied in the course description.**

OR

= **Criterion is partially suggested in the course description.**

2

= **Criterion is clearly or strongly suggested by the course description.**

In a few cases there may be a need for a higher rating:

3

= **Course description requirements satisfy and go beyond the scope of this criterion reference.**

I. Students should solidify and increase mathematical knowledge and skills at and above the level of Algebra II or its equivalent.

- A. The course description indicates that opportunities will be provided for students to reinforce and increase their fluency with arithmetic and algebraic processes.
- B. The course description indicates that opportunities will be provided for continued experience with functions that include linear, quadratic, and exponential and possibly extend to some advanced functions, such as logarithmic, trigonometric, higher-degree polynomial, or piecewise-defined functions.
- C. The course description offers students new insight into mathematics by including topics from non-traditional areas—such areas might include finite or discrete mathematics, statistical reasoning and inference, computer applications, analytic geometry, non-Euclidean geometry, or geometric probability.

II. Students should deepen and enrich the ways they think about mathematics to elevate its study well beyond rote memorization to a process of analysis and interpretation that enables the learner to grapple with a range of complex questions, topics, and issues.

- A. The course description provides opportunities for students to think conceptually, in addition to procedurally, about mathematics.
- B. The course description requires students to justify approaches to and results of problems with compelling mathematical arguments and encourages the application of solid reasoning in multiple contexts and across disciplines.
- C. The course description provides students with opportunities to deepen their understanding of mathematical reasoning and supports the development of expertise in using the appropriate type of reasoning for a given situation.
- D. The course description encourages experimental thinking, inquisitiveness, and evaluation of problem solving processes.
- E. The course description includes situations that engage students in the use of abstraction and generalization.

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= Course description requirements satisfy and go beyond the scope of this criterion reference.

- F. The course description includes opportunities to see connections among the branches of mathematics.
- G. The course description provides opportunities for the effective use of modern technologies, such as graphing and algebraic calculators or software, data gathering probes, or computer-assisted sampling.

III. Students should develop an appreciation for and experience with a variety of applications of mathematics across disciplines and in practical situations.

- A. The course description includes a focus on solving non-routine problems that are of interest to students who do not currently plan to follow a mathematics-intensive postsecondary pathway in college or work.
- B. The course description provides opportunities for students to determine the key elements of a problem, translate them into related mathematics, apply relevant mathematical strategies to solve problems, and communicate results using terminology that is understandable, correct, and appropriate for the situation.
- C. The course description provides students with problems that can be solved in more than one way and with the opportunity to evaluate the effectiveness and efficiency of particular solution methods.
- D. The course description encourages student persistence when solving problems, including those problems that require extended time and/or the gathering of information for their solutions.
- E. The course description requires students to visually represent a situation using mathematical diagrams or representations and to apply common sense to evaluate the reasonableness of solutions for practical problems in terms of the context.

The 15 criteria included as part of this evaluative tool were found to be generally present in effective capstone courses. Using this tool, which includes these criteria and the accompanying rating scale, evaluators can gain insight into the possible effectiveness of a course. A template, which allows curriculum developers and evaluators to determine an average score for a course curriculum description, is provided as an additional tool for judging the effectiveness of one or more courses. Promising courses would have an average score that approaches 2 and should demonstrate consistency across most criteria. (Ratings of 3 are extremely rare in these courses and generally are found only in very focused and specific course descriptions.

For example, computer math courses generally rated 3s in the technology criterion – II.G.)

In using these criteria to evaluate a number of capstone courses, including those courses identified below as strong, Achieve was able also to get a sense of the criteria themselves. Reviewers found the strongest representations of the criteria in these effective courses to be in the areas of non-traditional topics (I.C), use of technology (II.G), and non-routine problems (III.A). The criteria with the weakest representation, when judged against these courses, were algebraic function review (II.B), the use of abstraction and generalization (II.E), connections among the branches of mathematics (II.F), and problems with multiple possible solutions (III.C). This group of criteria received an average score of around 1.5, slightly lower than others. Capstone course designers might use this information when thinking about topics to be included in effective course curricula.

Examples of Alternative Capstone Courses

Below is an illustrative, though not exhaustive, list of some alternative capstone courses that are being used in various states, districts, and schools around the United States. All of the courses included here scored within an acceptable range (1.5 to 2.0) when reviewed using our 15 criteria. Those offering the widest variety of topics and most extensive descriptions fared best in this review. However, those few with narrower scopes or less detail in their descriptions still show promise as high-interest courses for fourth-year high school students who plan to study or work in fields that are not necessarily mathematically oriented. The courses that did not make this list tended to be repeated overviews of a typical Algebra I and II curriculum. While such review may be seen as necessary, it is advised that the review come in a instructional form different from the one already experienced by the student. These students crave relevance in their mathematical studies and are more likely to thrive in a course that applies to their lives and interests.

- **International Baccalaureate's *Mathematical Studies Standard Level*** (available online to IB school personnel; www.online.ibo.org)

This international curriculum includes a strong review of algebraic processes and provides an opportunity for students to statistically investigate a topic of their choosing in depth. It is targeted at students with varied backgrounds and abilities. More specifically, it is designed to build confidence and encourage an appreciation of mathematics in students who do not anticipate a need for mathematics in their future studies. Students taking this course need to be already equipped with fundamental skills and a rudimentary knowledge of basic processes. The course concentrates on mathematics that can be applied to contexts related to other subjects being studied and common real-world occurrences. Students must produce a project—a piece of written work based on personal research, guided and supervised by the teacher. The project provides an opportunity for students to carry out a mathematical investigation in the context of another course being studied or another interest, using skills learned before and during the course.

The students most likely to select this course are those whose main interests lie outside the field of mathematics, and for many students this course will be their final experience of being taught formal

mathematics. All parts of the syllabus have therefore been carefully selected to ensure that an approach starting with first principles can be used. As a consequence, students can use their inherent logical thinking skills and do not need to rely on standard algorithms and remembered formulas. Because of the nature of this course, teachers may find that less formal, shared learning techniques can be more stimulating and rewarding for students. Lessons that use an inquiry-based approach, starting with practical investigations where possible, followed by analysis of results and leading to the understanding of a mathematical principle and its formulation into mathematical language are often most successful in engaging the interest of students. Furthermore, this type of approach is likely to assist students in their understanding of mathematics by providing a meaningful context and by leading them to understand more fully how to structure their work for the project.

- **Advanced Placement Computer Science A** (www.collegeboard.com/student/testing/ap/sub_compscia.html)

AP Computer Science A builds upon a foundation of mathematical reasoning that should be acquired before attempting the course. Students should be able to design and implement computer-based solutions to problems in a variety of application areas, implement commonly used algorithms and data structures, and develop and select appropriate algorithms and data structures to solve problems. The necessary prerequisites for entering AP Computer Science A include knowledge of basic algebra and experience in problem solving. A student in the course should be comfortable with functions and the concepts found in the uses of functional notation.

A large part of this course is built around the development of computer programs that correctly solve a given problem. These programs should be understandable, adaptable, and, when appropriate, reusable. At the same time, the design and implementation of computer programs is used as a context for introducing other important aspects of computer science, including the development and analysis of algorithms, the development and use of fundamental data structures, the study of standard algorithms and typical applications, and the use of logic and formal methods.

The goals of an AP course in computer science are comparable to those in the introductory sequence of courses for computer science majors offered in college and university computer science departments. It is not expected or intended, however, that all students in the AP Computer Science A course will major in computer science at the university level. AP Computer Science A is intended to serve both as an introductory course for computer science majors and as a course for people who will major in other disciplines that require significant involvement with technology. It is not a substitute for the usual college-preparatory mathematics courses.

- **Advanced Placement Statistics** (www.collegeboard.com/student/testing/ap/sub_stats.html)

The Advanced Placement Program offers a course and exam in statistics to secondary school students who wish to complete studies equivalent to a one-semester, introductory, non-calculus-based college course in statistics concepts and tools for collecting, analyzing, and drawing conclusions from data. Students are exposed to four broad conceptual themes: Exploring Data (describing patterns and departures from patterns), Sampling and Experimentation (planning and conducting a study), Anticipating Patterns (exploring random phenomena using probability and simulation), and Statistical Inference (estimating population parameters and testing hypotheses).

In this course, students are expected to construct their own knowledge by working individually or in small groups to plan and perform data collection and analyses while the teacher serves in the role of a consultant, rather than as a director. This approach gives students ample opportunities to think through problems, make decisions, and share questions and conclusions with other students as well as with the teacher. Important components of the course include, as part of the concept-oriented instruction and assessment, the use of technology, projects and laboratories, cooperative group problem solving, and writing. This approach to teaching AP Statistics will allow students to build interdisciplinary connections with other subjects and with their world outside school. Students who successfully complete the course and exam may receive credit for a one-semester introductory college statistics course.

The AP Statistics course depends heavily on the availability of technology suitable for the interactive, investigative aspects of data analysis. Therefore, schools should make every effort to provide students and teachers easy access to graphing calculators and/or computers to facilitate the teaching and learning of statistics.

- **Advanced Functions and Modeling** (NC) (www.ncpublicschools.org/curriculum/mathematics/scos/2003/9-12/56advancedfunctions)

Content for this course consists of situations that generate data, which are then modeled using elementary (linear, quadratic, exponential) or more advanced (polynomial, logarithmic, trigonometric) functions and their transformations. Students are expected to evaluate the goodness of fit of such models and use them to make predictions, understanding the power and limitations of such predictions. This curriculum offers a review of key functions of algebra and encourages the use of technology. Non-traditional or extended topics are included. Prerequisites for the course include content typically found in courses in Geometry and Algebra II or their equivalents.

- **Advanced Mathematical Decision Making** (TX) (proposed) (www.utdanacenter.org/amdm)

Emphasizing practical and engaging problem solving through the use of technology in the context of functions and their applications, this course was designed to provide an appropriate fourth-year mathematics option for students planning to enter the workforce or a training program after high school or for students not intending to pursue a mathematics-intensive field in college. It may also provide a rich mathematics experience as an elective course for students following a pre-calculus/calculus path. Course topics are drawn from the areas of descriptive statistics, financial/economic literacy, and basic trigonometry. Students are expected to make informed decisions that make use of the underlying mathematics or statistics. A particular goal of the course is that students will formulate and solve problems and verify solutions in multiple ways, connecting mathematics to other disciplines and connecting mathematical topics with each other. Prerequisites include fluency in subject matter drawn from the Texas Essential Knowledge and Skills (TEKS) for Algebra I and Geometry, completion of Algebra II, and experience with mathematics drawn from authentic contexts.

- **Discrete mathematics** (VA) (www.doe.virginia.gov/VDOE/Instruction/Math/math_framework.html)

This course involves applications using discrete variables rather than continuous variables. It relies on modeling and understanding finite systems that are central to the development of the economy, the natural and physical sciences, and mathematics itself, to introduce the topics of social choice as a mathematical application, matrices and their uses, graph theory and its applications, and counting and finite probability. In addition, the processes of optimization, existence, and algorithm construction mathematics content develop sequentially in concert with a set of processes that are common to different bodies of mathematics knowledge. Students become mathematical problem solvers, communicate mathematically, reason mathematically, make mathematical connections, and use mathematical representations to model and interpret practical situations. Teachers help students make connections and build relationships among algebra, arithmetic, geometry, discrete mathematics, and probability and statistics. Connections can be made to other subject areas and fields of endeavor through applications. Using manipulatives, graphing calculators, and computer applications to develop concepts allows students to develop and attach meaning to abstract ideas. Topics include modeling and solving problems using vertex-edge graphs, problems in election theory and fair division, computer mathematics (including various sorting methods, coding systems, and Boolean logic), and recursion and optimization.

- **Probability and Statistics** (VA)
(www.doe.virginia.gov/VDOE/Instruction/Math/math_framework.html)

This course presents basic concepts and techniques for collecting and analyzing data, drawing conclusions, and making predictions. Applications may be drawn from a wide variety of disciplines ranging from the social sciences of psychology and sociology to education, health fields, business, economics, engineering, the humanities, the physical sciences, journalism, communications, and liberal arts. Students should be able to design an experiment, collect appropriate data, select and use statistical techniques to analyze the data, and develop and evaluate inferences based on the data. The mathematics content develops sequentially in concert with a set of processes that are common to different bodies of mathematics knowledge. Connections can be made to other subject areas and fields of endeavor through applications. Use of technology is imperative and should help students develop and attach meaning to abstract ideas. Students are encouraged to talk about mathematics, use the language and symbols of mathematics, communicate, discuss problems and problem solving, and develop competence and confidence in their mathematics skills. Content is drawn from the areas of descriptive statistics, data collection, probability, and inferential statistics.

- **Computer Mathematics** (VA)
(www.doe.virginia.gov/VDOE/Instruction/Math/math_framework.html)

This course provides students with experiences in using the computer to solve problems that can be set up as mathematical models. Students will develop and refine skills in logic, organization, and precise expression, thereby enhancing learning in other disciplines. Programming is introduced in the context of mathematical concepts and problem solving. Students will define a problem; develop, refine, and implement a plan; and test and revise the solution. Content includes applications in

number sense, algebraic functions, graphics, and coordinate geometry. Teachers should help students make connections and build relationships among algebra, arithmetic, geometry, discrete mathematics, and probability and statistics. Connections can be made to other subject areas and fields of endeavor through applications. Throughout the study of mathematics, students should be encouraged to talk about mathematics, use the language and symbols of mathematics, communicate, discuss problems and problem solving, and develop their competence and confidence in their mathematics skills.

- **Discrete Mathematics** (IN) (www.doe.state.in.us/standards/docs-Math/2006-Math-DiscreteMath.pdf)

This course consists of units in counting techniques, matrices, recursion, graph theory, social choice, linear programming, and game theory. Mathematical reasoning and problem solving are emphasized along with connections to other branches of mathematics and other disciplines. The course develops students' ability to read, write, listen, ask questions, think, and communicate about math to deepen their understanding of mathematical concepts. Proper communication using the language of mathematics—its terminology, symbols, formulas, graphics, displays, etc.—is a dynamic tool for solving problems and communicating and expressing mathematical ideas

- **Probability and Statistics** (IN) (www.doe.state.in.us/standards/docs-Math/2006-Math-ProbStat.pdf)

This course consists of units in descriptive statistics, probability, and statistical inference. Woven throughout Indiana's mathematics standards are the following processes: mathematical reasoning and problem solving, communication, representation, and connections. The ability to read, write, listen, ask questions, think, and communicate about math develops and deepens students' understanding of mathematical concepts. Students are expected to read text, data, tables, and graphs with comprehension and understanding. Their writing should be detailed and coherent, and they should use correct mathematical vocabulary. Students are expected to explain answers, justify mathematical reasoning, and describe problem solving strategies.

- **Modeling and Quantitative Reasoning** (OH) (www.ohiorc.org/pm/math/math_pmc.aspx)

This course prepares students to investigate contemporary issues mathematically and apply the mathematics learned in earlier courses to answer questions that are relevant to their civic and personal lives. It reinforces student understanding of percentages, functions and their graphs, probability and statistics, and multiple representations of data and data analysis. It also introduces functions of two variables and graphs in three dimensions. The applications provide an opportunity for students to gain a deeper understanding and expand upon material from previous math instruction. This course also shows the connections between the branches of mathematics and the various applications of mathematics. Today, numbers and data are critical parts of public and private decision making. Decisions about health care, finances, science policy, and the environment are decisions that require citizens to understand information presented in numerical form and in tables, diagrams, and graphs. Students must develop skills to analyze complex issues using quantitative tools. In addition to a textbook, teachers are advised to use online resources, newspapers, and magazines to identify problems that are appropriate for the course. Students should be encouraged to

find issues that can be represented in a quantitative way and shape them for investigation. Appropriate use of available technology is essential as students explore quantitative ways of representing and presenting the results of their investigations.

- **Computer Mathematics** (AR) (arkansased.org/teachers/pdf/computer_mathematics.pdf)

This course provides students with experiences in using the computer to solve problems that can be set up as mathematical models. Students entering this course must be currently enrolled in or have already taken Algebra II. Students will develop and refine skills in logic, organization, and precise expression, thereby enhancing learning in other disciplines. Programming is introduced in the context of mathematical concepts and problem solving. Students will define a problem; develop, refine, and implement a plan; and test and revise the solution. It is recommended that class size be no larger than 20 students because of the hands-on projects and activity nature of the course. Teachers help students make connections to other subject areas and fields of endeavor through applications. Use of manipulatives, graphing calculators, and computer spread sheet applications helps students develop and attach meaning to abstract ideas.

- **Data Analysis** (CA) (www.ucop.edu/a-gGuide/ag/course_descriptions/courses.php)

This course covers the basic principles of descriptive statistics, exploratory data analysis, design of experiments, sampling distributions and estimation, and fitting models to data. Statistical concepts are studied in order to understand related methods and their applications. Other topics include probability distributions, sampling techniques, binomial distributions, and experimental design. The course also looks extensively at the principles of hypothesis testing and statistical inference. Measuring the probability of an event, interpreting probability, and using probability in decision making are central themes of this course. Examples from games of chance, business, medicine, policymaking, the natural and social sciences, and sports will be explored. Use of computers and graphing calculators exposes students to the power and simplicity of statistical software for data analysis. The Texas Instruments TI-83+ is used extensively as a learning tool and is required for the course. Prerequisite: full year of Algebra II with passing grade.