

Rolling for the Big One

A-E Strand(s): Probability/Statistics. Sample Courses: Middle School Course 1, Middle School One-Year Advanced Course, Integrated 1, and Geometry.

Topic/Expectation

PS.B.1 Compound probability

- a. Calculate the probabilities of compound events.

Other Topic/Expectation

PS.A.1 Simple probability

- c. Compare probabilities of two or more events and recognize when certain events are equally likely.

Rationale

Students at this level are beginning to deal with more than one event at a time. This task helps students understand the different types of probabilities related to more than one event.

Instructional Task

Grace plays a game with her nephew, Tim, in which they try to build the largest seven-digit number possible with randomly rolled numbers. Each player has a game board as shown:

_ , _ _ _ , _ _ _

Players take turns rolling a 10-sided polyhedron, each side labeled with a different digit from 0 through 9. On his or her turn, the player writes the number that comes up on the roll in one space on his or her game board. Once the digit is written, it cannot be moved. So far in the game, Tim's game board is:

_ , 76_ , 412

Grace's game board is:

_ , _66, _30

On her fifth turn, Grace rolls a 7. She must decide where to place the 7. She thinks 7 is a pretty large number, but she wonders whether she might roll a 7, 8, or 9 on her two remaining rolls. She decides to use the probability of rolling a number at least as large as 7 on her next two rolls to help her choose where to place the 7 she just rolled on her fifth turn. If the probability of rolling a 7, 8, or 9 is greater than $\frac{1}{2}$, Grace will put the 7 in the hundred thousands place; otherwise, she will put it in the millions place. Grace needs to calculate the probability of rolling a 7, 8, or 9 on either the 6th or 7th roll, written as $P(7, 8, \text{ or } 9 \text{ on the 6th roll or } 7, 8, \text{ or } 9 \text{ on the 7th roll})$.

1. Grace finds:
 - a. $P(\text{rolling a 7 on any single roll}) = 1/10$. Why is this true?
 - b. $P(\text{rolling a 7, 8, or 9 on any single roll}) = 3/10$. Why is this true?
 - c. $P(\text{rolling a number less than 7 on any single roll}) = 7/10$. Why is this true?
 - d. If $P(A)$ = the probability of rolling a 7, 8, or 9 on the 6th roll, and $P(B)$ = the probability of rolling a 7, 8, or 9 on the 7th roll, then $P(A \text{ or } B)$ is not the same as $P(A) + P(B)$. Why not?
 - e. $P(\text{rolling 7, 8, or 9 on 6th roll or 7, 8, or 9 on 7th roll}) = 1 - P(\text{both the 6th and 7th rolls are less than 7})$. Why is this true?
2. Is the outcome on the 7th roll independent or dependent of the result of the 6th roll? Why or why not?
3. What is $P(\text{both the 6th and 7th rolls are less than 7})$?
4. What is $P(\text{rolling 7, 8, or 9 on 6th roll or 7, 8, or 9 on 7th roll})$?
5. Where should Grace place the 7 she rolled on her 5th turn?

Discussion/Guiding Questions/Extensions

1. Ask students to play the game once before answering the questions so they have some understanding of it, thinking about how likely it is they will roll a higher number later in the game.
2. Then, ask students to answer question 1. Have students share their responses and justifications.
3. Ask students to answer all parts of questions 2 -5. Summarize student responses to those questions, then return to the question in Part 1 and ask, “How do these other questions (and answers) relate to the original question?” and “How does knowing these other probabilities and how they are calculated help Grace make choices about where to place a 7 on the 5th roll?”

Sample Solutions

1. Grace finds:
 - a. $P(\text{rolling a 7 on any single roll}) = 1/10$. Why is this true?

$P(\text{rolling a 7 on any single roll}) = 1/10$ because there are 10 possible digits that could be rolled with equal likelihood, one of which is a 7.

- b. $P(\text{rolling a 7, 8, or 9 on any single roll}) = 3/10$. Why is this true?

$P(\text{rolling a 7, 8, or 9 on any single roll}) = 3/10$, since any of 3 possibilities are designated from the 10 that are possible.

$P(\text{rolling 7, 8, or 9 on any single roll}) = P(\text{rolling a 7}) + P(\text{rolling an 8}) + P(\text{rolling a 9}) = 1/10 + 1/10 + 1/10 = 3/10$.

[Teacher note: You are equally likely to roll a 7, 8, or 9. These events are exclusive because they cannot happen at the same time. That is, if you roll a 7 you cannot roll an 8 or 9 at the same time.]

- c. $P(\text{rolling a number less than 7 on any single roll}) = 7/10$. Why is this true?

$P(\text{rolling a number less than 7 on any single roll}) = 1 - 3/10 = 7/10$

Students might also respond $P(0, 1, 2, 3, 4, 5, 6) = 7/10$, representing 7 specified outcomes of 10 possible outcomes.

[Teacher note: The event of rolling a number less than 7 is the complement of the event of rolling a 7, 8, or 9, since it includes all the possible outcomes not addressed in the second event.]

- d. If $P(A)$ = the probability of rolling a 7, 8, or 9 on the 6th roll, and

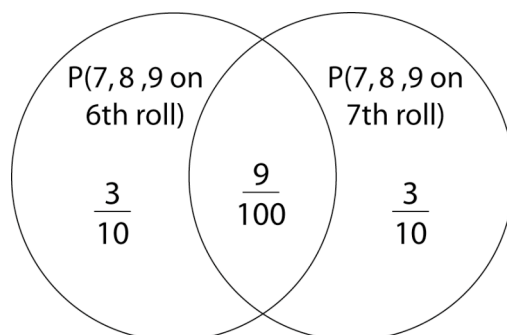
$P(B)$ = the probability of rolling a 7, 8, or 9 on the 7th roll, then

$P(A \text{ or } B)$ is not the same as $P(A) + P(B)$. Why not?

$P(A \text{ or } B) \neq P(A) + P(B)$ because $P(A) + P(B)$ counts the probability that a 7, 8, or 9 is rolled both times [$P(A \text{ and } B)$] twice.

So, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$$P(A \text{ or } B) = \frac{3}{10} + \frac{3}{10} - \frac{9}{100} = \frac{51}{100} = .51$$



- e. $P(\text{rolling 7, 8, or 9 on 6th roll or 7, 8, or 9 on 7th roll}) = 1 - P(\text{both the 6th and 7th rolls are less than 7})$. Why is this true?

$P(\text{rolling } 7, 8, \text{ or } 9 \text{ on } 6\text{th roll } \text{or } 7, 8, \text{ or } 9 \text{ on } 7\text{th roll}) = 1 - P(\text{rolling } 0, 1, 2, 3, 4, 5, \text{ or } 6 \text{ on both the } 6\text{th and } 7\text{th rolls})$ since rolling a 7, 8, or 9 on at least one roll is the complement of not rolling a 7, 8, or 9 at least once.

2. Is the outcome on the 7th roll independent or dependent of the result of the 6th roll? Why or why not?

The outcome on the 7th roll is independent of the result of the 6th roll because the outcome on the 6th roll doesn't affect outcomes on the last roll. That is, since the outcome of rolling any number on the 6th roll is $1/10$, and the outcome of rolling any number on the 7th roll is $1/10$, they are separate events that do not depend on each other. Each roll carries the same possibilities as any other roll.

3. What is $P(\text{both the } 6\text{th and } 7\text{th rolls are less than } 7)$?

$P(\text{both the } 6\text{th and } 7\text{th rolls are less than } 7) = P(\text{rolling a number less than } 7 \text{ on } 6\text{th roll } \text{and} \text{ rolling a number less than } 7 \text{ on } 7\text{th roll}) = P(\text{rolling a number less than } 7 \text{ on } 6\text{th roll}) \bullet P(\text{rolling a number less than } 7 \text{ on } 7\text{th roll}) = (0.7)(0.7) = 0.49.$

4. What is $P(\text{rolling } 7, 8, \text{ or } 9 \text{ on } 6\text{th roll } \text{or } 7, 8, \text{ or } 9 \text{ on } 7\text{th roll})$?

Using the answer to question 1.d above, $P(\text{rolling } 7, 8, \text{ or } 9 \text{ on } 6\text{th roll } \text{or } 7, 8, \text{ or } 9 \text{ on } 7\text{th roll}) = 1 - P(\text{both the } 6\text{th and } 7\text{th rolls are less than } 7) = 1 - 0.49 = 0.51.$ Or, $3/10 + 3/10 - (3/10 \bullet 3/10) = (6/10 - 9/100) = 51/100 = 0.51.$

5. Where should Grace place the 7 she rolled on her 5th turn?

Grace should place the 7 in the hundred thousands place since the probability that she will get a roll at least as high as this before the end of the game is 0.51, which is greater than $1/2$. That is, she is more likely than not to roll a 7, 8, or 9 on her 6th or 7th roll.