

Regional Triangles

A-E Strand: Geometry. Sample Courses: Middle School Course 1, Middle School One-Year Advanced Course, Integrated 1, Integrated 3, Algebra 1, and Geometry.

Topic/Expectation

G.C.1 Geometry of a circle

- c. Recognize, verify, and apply statements about the relationships between central angles, inscribed angles, and the circumference arcs they define.
- e. Determine the length of line segments and arcs, the size of angles, and the area of shapes that they define in complex geometric drawings.

G.C.2 Axioms, theorems, proofs in geometry

- b. Present and analyze direct and indirect geometric proofs using paragraphs, two-column, or flow-chart formats.

G.A.1 Angles and triangles

- a. Know the definitions and basic properties of angles and triangles in the plane and use them to solve problems.

D.C.2 Mathematical reasoning

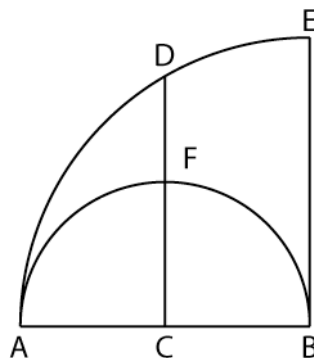
- d. Make, test, and confirm or refute conjecture using a variety of methods.

Rationale

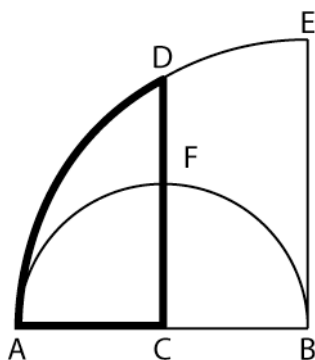
This task may be used with students who have various levels of geometric understanding to emphasize geometric vocabulary, apply basic theorems involving circles, make conjectures, and justify or prove conjectures.

Instructional Task

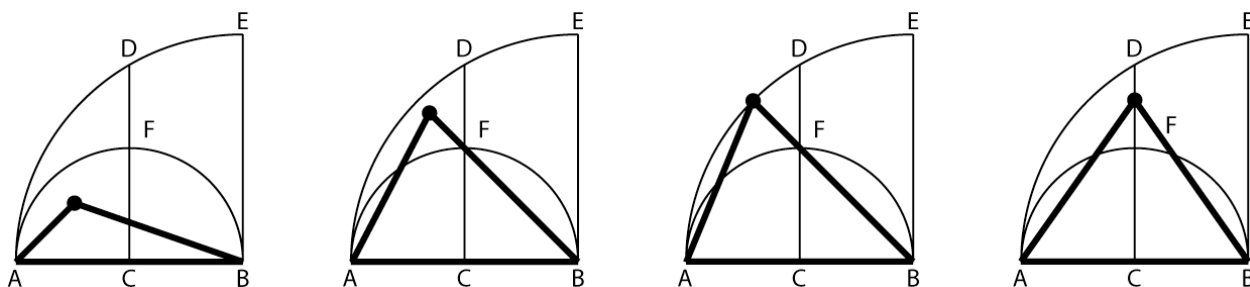
Segment AB, shown in the figure below, is the diameter of a semicircle and the radius of a quarter circle. Segment DC is the perpendicular bisector of segment AB.



Segment AB will be the longest side of a triangle with the third vertex lying in the region bounded by segments AC, CD, and arc AD, seen in bold below.



Some examples of possible triangles that fit these criteria are shown below.



Make conjectures about what types of triangles are formed based upon the region where the third vertex is located. Then use geometric reasoning to verify the conjectures.

Discussion/Further Questions/Extensions

Teachers may wish to provide students with the table found in the solution section that has only the first column filled in. Novice students can make conjectures and test these by using technology resources, paper folding, or construction tools. Students at all levels can gain experience with geometric relationships by constructing a picture and emphasizing key vocabulary such as radius, diameter, perpendicular bisectors, arcs, and sectors. More experienced students can justify their conjectures using formal proofs, possibly including transformations.

Sample Solutions

The different regions can be described and conjectures formed for each region as seen below. Reasoning and justification will vary according to the student's level of experience with geometry and can take many forms. For example, students can explore and reason using transformations, technology software, construction techniques, etc., or they may use formal proofs.

Students might use technology to explore and find patterns of various point placements.

Region of 3rd vertex	Type of triangle formed	Reasoning and justification (samples)
At point D	Equilateral	Segments AB and BD are both radii of circle B and are congruent. Since segment CD is a perpendicular bisector of segment AB, AD is a reflection of BD, so segments AD and BD are congruent. Since all three sides of $\triangle ADB$ are congruent, it is equilateral.
Between points A and D on arc AD	Acute Isosceles	Any point X on arc AD, together with B, forms a radius of circle B. Since all radii of a circle are congruent, segments BX and BA are congruent, making the triangle an isosceles triangle. The base angles of an isosceles triangle are congruent and acute. The vertex angle ABX is also acute since the angle is a central angle whose measure is less than the measure of angle ABE, which is a right angle.
Between points A and F on arc AF	Right	The measure of an inscribed angle of a circle is one-half the measure of its intercepted arc. The intercepted arc is a semicircle (180°), so an angle with vertex on arc AF is a right angle.
At point F	Right Isosceles	At point F, the new triangle is a right triangle (see above). Since F lies on the perpendicular bisector of AB, segments BF and AF are congruent; therefore, the triangle is also isosceles.
Interior of region bounded by segment DF and arcs AF and AD	Acute scalene	Since the angle is formed from the intersection of secant lines, the angle is one-half the absolute value of the difference of the measures of the semicircle (180°) and the smaller intercepted arc, making the angle less than 90° . Since the vertex is not on the perpendicular bisector of diameter AB, the triangle formed will not be isosceles.
Between points D and F on segment DF	Acute isosceles	The triangle is isosceles since X is on the perpendicular bisector of AB. See the above explanation for why the triangle is acute.

Region of 3rd vertex	Type of triangle formed	Reasoning and justification (samples)
Interior of sector ACF	Obtuse scalene	By extending the sides of the angle to make chords, vertical angles are formed. The measure of one of the vertical angles is half the sum of the measure of the semicircle (180°) and the other intercepted arc, making the angle more than 90° . Since the vertex is not on the perpendicular bisector of the diameter AB, the triangle formed will not be isosceles.
Between points F and C on segment FC	Obtuse isosceles	The triangle is isosceles since X is on the perpendicular bisector of AB. If X is between F and C, angle AXB is obtuse (see the explanation for obtuse scalene above).