Neighborhood Park

A-E Strand(s): Geometry. Sample Courses: Middle School Course 1, Middle School Course 2, Middle School One-Year Advanced Course, Integrated 1, and Geometry.

Topic/Expectation

G.A.4 Length, area and volume

a. Identify and distinguish among measures of length, area, surface area and volume.
b. Calculate the perimeter and area of triangles, quadrilaterals and shapes that can be decomposed into triangles and quadrilaterals that do not overlap.

G.B.5 Scaling, dilation and dimension

a. Analyze and represent the effects of multiplying the linear dimensions of an object in the plane or in space by a constant scale factor, *r*.
b. Describe the effect of a scale factor *r* on length, area and volume.
d. Interpret and represent origin-centered dilations of objects on the coordinate plane.

Other Topic/Expectation(s)

N.B.2 Exponents and roots

d. Convert between forms of numerical expressions involving roots and perform operations on numbers expressed in radical form.

G.B.2 Coordinates and slope

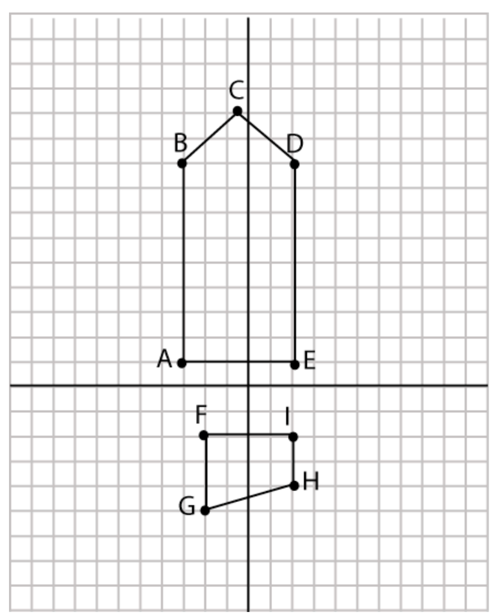
a. Represent and interpret points, lines and two-dimensional geometric objects in a coordinate plane.

Rationale

In this task, students will investigate the effects of a scale factor, *r*, on length, area and volume in a problem-solving context.

Instructional Task

A new neighborhood is being developed, and Raul is in charge of the team that is building a community swimming pool and playground. Raul initially hired Best Sports Design Company to develop a plan for the design and location of the swimming pool and playground. (The plans are shown on the coordinate grid to the right.) The company's designer mistakenly drew plans for a much smaller backyard pool and play area.



Raul has decided to hire you to help fix the design company's plans and has provided you with the following guidelines:

a. The plans need to be scaled up using a geometric model involving two dilations with centers at the point (0, 0), as shown on the design below.

b. The dilation for the pool will have a scale factor of 5 and the dilation for the playground will have a scale factor of 3.

A geometric model can be used to determine:

- the exact location for the new swimming pool and playground area,
- the total length of fencing material needed to create a fence around each new area,
- the total amount of material needed to build a protective cover over each area during the construction phase,
- the total volume of water needed to fill the new swimming pool to a given depth, and
- the total quantity of rubber chips needed to build a base of a given depth in the new playground area.

1. Show the location of the new swimming pool and the new playground area on the drawing below. (In the drawing, 1 square = 1 yd^2.)

2. Complete the information in the table below and then use the table to help answer the questions that follow. In the "comparison" column, describe the relationship between the original and the new measurements.

Figure	Original	New	Comparison
Pool side lengths and depth (see note below)			
Playground side lengths and depth (see note below)			
Pool perimeter			
Playground perimeter			
Area of pool			
Area of playground			
Volume of water for pool			
Volume of rubber chips for playground			

(Note: The new pool has dimensions that are five times greater than the original pool. The original pool was designed to have a depth of 1 yard, so the new pool will have a depth of 5 yards. The new playground has dimensions that are three times greater than the original playground. The original playground was designed to have a depth of 0.5 yards, so the new playground will have a depth of 1.5 yards.)

3. Use the information in the table to help you answer a–c below. Show support for each answer.
   a. What happens to the length and perimeter of an object in the plane if its linear dimensions are scaled by a constant scale factor, *r*?
   b. What happens to the area of an object in the plane if its linear dimensions are scaled by a constant scale factor, *r*?
   c. What happens to the volume of an object in the plane if its linear dimensions and depth are scaled by a constant scale factor, *r*?

4. What is the total length of fencing material needed to build fences around both new areas?

5. A rectangular tarp will be needed to cover the pool and another to cover the playground during construction. What are the total areas of the pool and playground that need to be covered? Explain how you found each answer. Since each tarp is rectangular in shape, what should be the required minimum dimensions of the tarp that will cover the entire pool and of the tarp that will cover the entire playground? How much extra area (that is, area that is not directly covering the pool or playground) will each tarp cover?

6. How many cubic yards of water will be needed to fill the pool? Explain how you found your answer. Is it reasonable to scale the depth of the pool by a factor of 5? Why or why not?

7. How many cubic yards of rubber chips are needed to fill the play area? Explain how you found your answer. Is it reasonable to scale the depth of the rubber chips by a factor of 3? Why or why not?

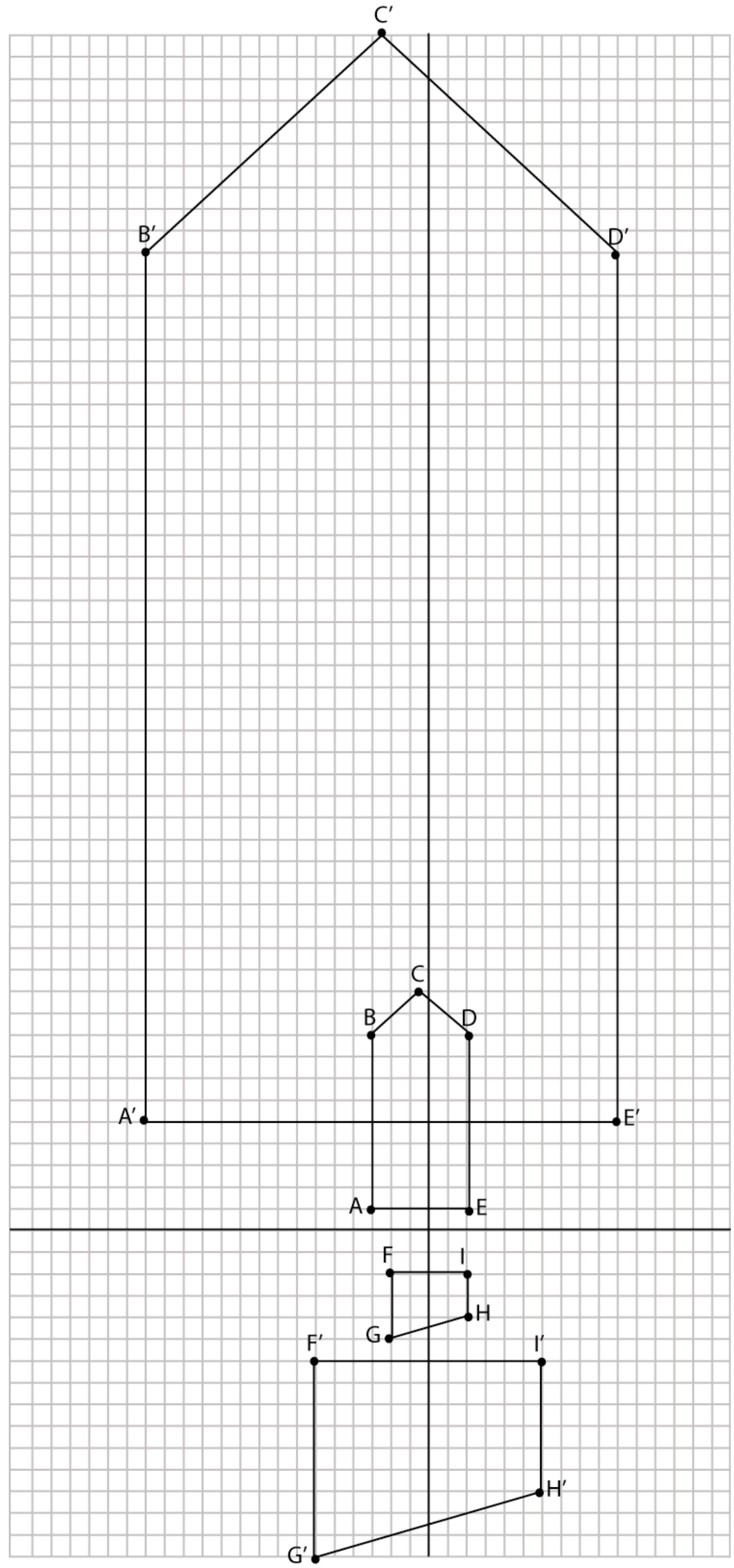
8. If you had only 20 linear yards of fencing material for the playground, what is the greatest scale factor you could use to dilate the original playground with a center (0,0) and still have enough fencing to surround it?

Discussion/Guiding Questions/Extensions

The use of dilations is a special case of similarity and proportionality. After exploring the dilation problems in depth, students should have opportunities to compare and contrast this situation with other uses of similarity and proportionality in different contexts.

Sample Solutions

1. Show the location of the new swimming pool and the new playground area on the drawing below. (In the drawing, 1 square = 1 yd^2.)


C′
B′
D′
C
B
D
A′
E′
A
E
F
I
H
G
F′
I′
H′
G′

2. Complete the information in the table below and then use the table to help answer the questions that follow. In the "comparison" column, describe the relationship between the original and the new measurements.

Figure	Original	New	Comparison
Pool side lengths and depth (see note below)	*AB = 8 yd*	*A'B' = 40 yd*	*Each new side is five times longer than the corresponding original side.*
	$BC = \sqrt{10.25} \approx 3.2$ *yd*	*B'C'* $\approx$ *16 yd*	
	CD $\approx$ *3.2 yd*	*C'D'* $\approx$ *16 yd*	
	DE = 8 yd	*D'E' = 40 yd*	
	AE = 5 yd	*A'E' = 25 yd*	
	Depth = 1 yd	*Depth = 5 yd*	
Playground side lengths and depth (see note below)	*FG = 3 yd*	*F'G' = 9 yd*	*Each new side is three times longer than the corresponding original side.*
	$GH = \sqrt{17} \approx 4.1$ *yd*	$G'H' = 3\sqrt{17} \approx 12.4$ *yd*	
	HI = 2 yd	*H'I' = 6 yd*	
	IF = 4 yd	*I'F' = 12 yd*	
	Depth = 0.5 yd	*Depth = 1.5 yd*	
Pool perimeter	$21 + 2\sqrt{10.25} \approx$ *27.4 yd*	$105 + 2\sqrt{256.25} \approx$ *137.0 yd*	*The new perimeter is 5 times greater than the original perimeter.*
Playground perimeter	$9 + \sqrt{17} \approx 13.1$ *yd*	$27 + 3\sqrt{17} \approx 39.4$ *yd*	*The new perimeter is 3 times greater than the original perimeter.*
Area of pool	*45* yd^2	*1125* yd^2	*The new area is 25 times greater than the original area.*
Area of playground	*10* yd^2	*90* yd^2	*The new area is 9 times greater than the original area.*
Volume of water for pool	*45* yd^3	*5625* yd^3	*The new volume is 125 times greater than the original volume.*
Volume of rubber chips for playground	*5* yd^3	*135* yd^3	*The new volume is 27 times greater than the original volume.*

(Note: The new pool has dimensions that are five times greater than the original pool. The original pool was designed to have a depth of 1 yard, so the new pool will have a depth of 5 yards. The new playground has dimensions that are three times greater than the original playground. The original playground was designed to have a depth of 0.5 yards, so the new playground will have a depth of 1.5 yards.)

3. Use the information in the table to help you answer a–c below. Show support for each answer.

   a. What happens to the length and perimeter of an object in the plane if its linear dimensions are scaled by a constant scale factor, *r*?

      Multiplying the linear dimensions of an object in a plane by a constant scale factor, r, increases the linear dimensions of the new object by r times. This is illustrated in the chart above—each new pool dimension is five times greater than the corresponding dimension of the original pool, and each new playground dimension is three times greater than the corresponding dimension of the original playground. In addition, the same relationship exists for the perimeters of each object. That is, multiplying the linear dimensions of an object in a plane by a constant scale factor, r, increases the perimeter of the new object by r times.

   b. What happens to the area of an object in the plane if its linear dimensions are scaled by a constant scale factor, *r*?

      Multiplying the linear dimensions of an object in a plane by a constant scale factor, r, increases the area of the new figure by r^2*. For example, with a scale factor of 5, the area of the new pool is 25 times the area of the original pool, and with a scale factor of 3, the area of the new playground is 9 times the area of the original playground.*

   c. What happens to the volume of an object in the plane if its linear dimensions and depth are scaled by a constant scale factor, *r*?

      Multiplying the linear dimensions and depth of an object in a plane by a constant scale factor, r, increases the volume of the new figure by r^3*. For example, with a scale factor of 5, the volume of the new pool is 125 times the volume of the original pool, and with a scale factor of 3, the volume of the new playground is 27 times the volume of the original playground.*

4. What is the total length of fencing material needed to build fences around both new areas?

   To find the total length of material needed to build the fences, add together the perimeter of the new pool and the perimeter of the new playground. $105 + 2\sqrt{256.25} + 27 + 3\sqrt{17} \approx 176.4$ *yards of material is needed to build fences around the pool and playground.*

5. A rectangular tarp will be needed to cover the pool and another to cover the playground during construction. What are the total areas of the pool and playground that need to be covered? Explain how you found each answer. Since each tarp is rectangular in shape, what should be the required minimum dimensions of the tarp that will cover the entire pool and of the tarp that will cover the entire playground? How much extra area (that is, area that is not directly covering the pool or playground) will each tarp cover?

   To find the area of the new pool, divide the shape into a square and a triangle. Find the area of each figure, then find the sum of the two areas.

A (rectangle) = lw = (40yds x 25yds) = 1000 yds^2

A (triangle) = ½ bh = ½ (25yds x 10yds) = 125 yds^2

Total area = 1125 yds^2

(Students may choose to reference the calculation from the table.)

To find the area of the new playground, divide the shape into a rectangle and a triangle. Then find the area of each figure and add the two areas together to find the total area of the new playground.

A (rectangle) = lw = 12 yds x 6 yds = 72 yds^2

A (triangle) = ½ bh = ½ (3 yds x 12 yds) = 18 yds^2

Total area = 90 yds^2

The minimum dimensions for the tarp covering the pool area are 25 yds by 50 yds. The minimum dimensions for the tarp covering the playground are 9 yds by 12 yds.

*Since the area of the pool is 1125*yds^2*, a 25-yd by 50-yd tarp (which has a total area of 1250* yds^2*) will be large enough to cover the pool. The area the tarp will cover that is not part of the pool is 1250* yds^2 *– 1125* yds^2 *= 125*yds^2*.*

Since the area of the playground is 90 yds^2*, a 9-yd by 12-yd tarp (which has a total area of 108* yds^2*) will be large enough to cover the playground. The area the tarp will cover that is not part of the playground is 108* yds^2 *– 90* yds^2 *= 18* yds^2*.*

6. How many cubic yards of water will be needed to fill the pool? Explain how you found your answer. Is it reasonable to scale the depth of the pool by a factor of 5? Why or why not?

   To find the volume of the new pool, multiply the area of the base by the height (depth of the pool). Volume = Bh = 1125 yds^2 *x 5 yds =5625* yds^3*. (Note: B = area of the base.)*

   It is unreasonable to have a pool with a depth of 5 yards. A depth of 5 yards, which is equal to 15 feet, is too deep for a person to be able to stand in, and it would be very unusual for a neighborhood to build such a pool. In addition, most pools have a shallow end that is easy to stand in and a deeper end that people can dive into.

7. How many cubic yards of rubber chips are needed to fill the play area? Explain how you found your answer. Is it reasonable to scale the depth of the rubber chips by a factor of 3? Why or why not?

   *To find the volume of the new playground, multiply the area of the base by the height (depth). Volume = Bh = 90*yds^2 *x 1.5yds = 135* yds^3*.*

   It seems unreasonable to scale the depth in this example, which would increase the quantity of rubber chips needed. Most playground areas have only a thin layer of ground cover; just because a playground area gets larger does not mean that it should have a deeper layer of rubber chips.

8. If you had only 20 linear yards of fencing material for the playground, what is the greatest scale factor you could use to dilate the original playground with a center (0,0) and still have enough fencing to surround it?

   Since the original playground has a perimeter of $9+\sqrt{17} \approx 13.1$ *yds, a playground with a perimeter of 20 yds will have a scale factor equal to* $\dfrac{20\,yds}{9+\sqrt{17}\ yds} \approx 1.52$. *Therefore, the greatest scale factor that could be used is 1.52.*