Lesson 1, Part A  

Birth Dates and Personality Traits

In astrology, your sign is determined by your birthday. Some people believe that the 12 signs of the zodiac correspond to 12 personality types and that characteristics of your personality are related to which sign your birthday falls under. In this lesson, we will call this belief a sign theory and we will gather data to investigate its veracity (truthfulness or accuracy).

1) Do you think characteristics of your personality are related to your astrological sign? Why do you think so?

Objectives for the lesson

You will begin to understand:

☐ The process of stating statistical claims and its relation to the statistical decision-making process.

You will begin to be able to:

☐ Evaluate the strength of evidence against a claim about a population proportion.

In the questions below, you will investigate whether personality characteristics are truly related to zodiac sign. The following table, “Astrology Investigation,” contains the zodiac signs with three lists of personality traits (Choice 1, Choice 2, and Choice 3). One of the three choices contains the descriptions written by astrologists for that sign; the other two choices contain descriptions of other signs.

2) Locate the sign that corresponds to your birthday. Look at the three lists of personality traits and pick the one that comes closest to describing you.

3) A student in another class, Alex, is skeptical of the sign theory and thinks that it is not correct (meaning, birth date has nothing to do with personality traits). If we assume that the theory is incorrect, as Alex thinks, will some classmates still pick the matching description? Explain why you think so.
4) To continue to investigate about the veracity of the sign theory and to try to convince Alex, we need to formulate a claim or hypothesis that aligns with his belief. Think about what fraction of the students in the class you expect to pick the astrologists’ description, still assuming that the theory is incorrect (i.e., choosing by chance alone). Complete the sentence below and explain your reasoning.

Claim: If the personality traits are not related to zodiac signs, then __/___ of the students in the class will correctly match personality traits to zodiac signs.

5) What fraction of the students in the class should pick the correct description (astrologists’ description) to convince Alex that, in fact, the sign theory is true? Explain.

6) Suppose that half the students in your class select the correct description. Does this guarantee that there is some truth to this theory? If not, suggest another explanation for why so many students picked the correct sign.

7) There are two possible explanations for why more than 1/3 of the class might choose the matching description:
   - The theory is correct, and the class data reflect that the theory is correct.
   - The theory is incorrect, and the class just got lucky, resulting in more than 1/3 picking the matching description.

Describe a strategy that might allow you to rule out the second explanation.
(Hint: Can you think of a way to generate new outcomes that could help you decide whether your class data, once you see them, are consistent with the “got lucky” explanation? Can you model a situation that you know is simply luck, to generate the outcomes that are possible?)
## Astrology Investigation

<table>
<thead>
<tr>
<th>Zodiac Sign</th>
<th>Choice 1</th>
<th>Choice 2</th>
<th>Choice 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>Strengths</strong></td>
<td><strong>Weaknesses</strong></td>
<td><strong>Strengths</strong></td>
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</tbody>
</table>
# Astrology Investigation (continued)

<table>
<thead>
<tr>
<th>Zodiac Sign</th>
<th>Choice 1</th>
<th>Choice 2</th>
<th>Choice 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>Strengths</strong></td>
<td><strong>Weaknesses</strong></td>
<td><strong>Strengths</strong></td>
</tr>
<tr>
<td><strong>Cancer (6/22 to 7/22)</strong></td>
<td>Emotional</td>
<td>Changeable</td>
<td>Patient</td>
</tr>
<tr>
<td></td>
<td>Loving</td>
<td>Moody</td>
<td>Reliable</td>
</tr>
<tr>
<td></td>
<td>Intuitive</td>
<td>Overemotional</td>
<td>Warmhearted</td>
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<td></td>
<td>Imaginative</td>
<td>Touchy</td>
<td>Loving</td>
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<td></td>
<td>Shrewd</td>
<td>Clinging</td>
<td>Persistent</td>
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<tr>
<td></td>
<td>Cautious</td>
<td>Unable to let go</td>
<td>Determined</td>
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<tr>
<td></td>
<td>Protective</td>
<td></td>
<td>Placid</td>
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<tr>
<td></td>
<td>Sympathetic</td>
<td></td>
<td>Security-loving</td>
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<tr>
<td><strong>Leo (7/23 to 8/21)</strong></td>
<td>Emotional</td>
<td>Changeable</td>
<td>Generous</td>
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<td></td>
<td>Loving</td>
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<td>Touchy</td>
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<td>Shrewd</td>
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<td>Cautious</td>
<td>Unable to let go</td>
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<tr>
<td></td>
<td>Sympathetic</td>
<td></td>
<td>Loving</td>
</tr>
<tr>
<td><strong>Virgo (8/22 to 9/23)</strong></td>
<td>Practical</td>
<td>Pessimistic</td>
<td>Nervous</td>
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<tr>
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<td>Prudent</td>
<td>Fatalistic</td>
<td>Tense</td>
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<td>Miserly</td>
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<td>Disciplined</td>
<td>Grudging</td>
<td>Inconsistent</td>
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<td></td>
<td>Patient</td>
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<td>Cunning</td>
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<td>Lively</td>
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<tr>
<td></td>
<td>Reserved</td>
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</table>
## Astrology Investigation (continued)

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<tr>
<td></td>
<td>Strengths</td>
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<td>Weaknesses</td>
<td>Strengths</td>
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<tr>
<td>Capricorn (12/23 to 1/20)</td>
<td>Practical</td>
<td>Pessimistic</td>
<td>Adventurous</td>
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<tr>
<td></td>
<td>Prudent</td>
<td>Fatalistic</td>
<td>Energetic</td>
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<td>Ambitious</td>
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<td>Pioneering</td>
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<tr>
<td></td>
<td>Reserved</td>
<td>Quick-witted</td>
<td>Quick-witted</td>
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<tr>
<td>Aquarius (1/21 to 2/19)</td>
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<tr>
<td>Pisces (2/20 to 3/20)</td>
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Lesson 1, Part A
Birth Dates and Personality Traits

Overview and student objectives

Overview
In this lesson, students explore the relationship between zodiac sign and personality traits. The overall goal is to give students a sense of what a statistical analysis is all about and to focus on the inferential process. Students may attempt to reach conclusions about a theory based on results gathered in class. Will the evidence be convincing enough to show that results are not simply chance?

In later tasks, students get more specific about how to decide whether data support a claim. The goal here is to expose students to the reasoning process and ideally to help them generate the reasoning process themselves.

Objectives
Students will begin to understand:

- The process of stating statistical claims and its relation to the statistical decision-making process.

Students will begin to be able to:

- Evaluate the strength of evidence against a claim about a population proportion.

Suggested resources and preparation

Materials and technology

- Computer, projector, document camera
- Student Pages for Lesson 1, Part A
- Coins and/or loaded dice (available via the internet)
- Arrive to class early so that the grouping process described in the Suggested instructional plan (next page) can begin promptly or even early. Arriving early will signal to the students that the class will be a warm but productive community where maximizing learning is the priority.
Prerequisite assumptions

Students should be able to:

- Recognize or be familiar with their zodiac sign.
- Compare rational numbers.

Making connections

This lesson:

- Connects back to the concept of probability and risk analysis from the *Foundations of Mathematical Reasoning* course.
- Connects forward to the need for collecting data to quantify how much evidence is enough to be convincing.
- Foreshadows the process of hypothesis testing, particularly in setting up a null and alternative hypothesis.

Background context

Students should have background knowledge of theories and what it means to formulate a claim or hypothesis related to the theory. In this lesson, they will engage in the process of gathering data to support or refute the claim or hypothesis.

Suggested instructional plan

Frame the lesson

(6 minutes)

**Student Page**

- Have students line up in order by birth month and day. Divide the number of students by 3 and then ask them to count off by that result. For example, 24/3 = 8. Counting off by 8’s will put the class in groups of 3. If additional students arrive, add them to an existing group to create groups of 4. (According to the theory of the lesson, this method will form groups with a variety of personalities. It also makes the selection of groups by chance.)

**Guiding Question**

- You may or may not feel comfortable asking students about their zodiac sign. Before distributing Student Pages for Lesson 1, Part A (or after asking students to find their copy), you may ask, for example, “Has anyone read their horoscope in the last week or so? Did it turn out to be correct?” Alternatively, you may prefer to have students answer question 1 privately on their paper.
• Display the Student Pages. Tell students they are going to investigate the relationship between birth date and personality traits.

• Transition to the lesson activities by briefly discussing the Objectives for the lesson.

Lesson activities
(15 minutes)

• Watch for students who are trying to fit their answers onto their Student Pages, and point out that the activity pages will not have sufficient room. Students should bring their own loose-leaf paper to insert between the activity pages in their binders, ensuring that they always have sufficient room to work without feeling the pressure to fill a certain space.

Assign questions 2–6.

• The type of reasoning required in questions 2–6 may be new to the students, so encourage their struggle and do not jump in with the right answer. Students may have different ways to approach these questions. You want to foster that and then discuss the different approaches later. (Note: Further discussion of this activity takes place in a later lesson.)

• Question 2: If students ask for the answers to the matching activity, tell them you will reveal those later. Right now, you want them to be comfortable with this matching process.

• Questions 3 and 4: You want students to realize that if there is no merit to this theory and that all students are simply guessing at random, then in the long run, about 1/3 of students will pick the correct description and 2/3 of students will pick the wrong one. The formulation of the claim is an introduction of stating a null hypothesis for future lessons. Watch for students who may be focusing on the two possibilities of matching or not matching the correct description. Those students may believe the long-run proportion would be 1/2, so they will need to be redirected to the three description choices. In addition, make sure students realize that if everyone is just guessing, then this fraction is predictable. Students may realize that they have to make this assumption across the board, not that some signs have a better match than others.

• Your class results may differ quite a bit from 1/3, especially if your class is quite small. In this case, you could discuss the idea of putting together the results from all of your classes. In Preview Assignment 1.C, students will explore running multiple trials.

• Question 5: This question may be a bit more difficult for students, and you may see a wide range of responses, which is fine. Some students will consider anything more than 1/3 of the class size;
others may want the entire class or just about the entire class. Let them know that this is a rather subjective question for now, and people may have different answers. Get students to consider whether there are outcomes above 1/3 but less than 1 that they would be surprised by. If an individual group is really struggling, ask group members to consider 30 or 40 coin tosses and whether there is some number of heads in a row they need to get before they begin to be suspicious that there is something funny going on. The point is that even in the face of randomness and chance, students can still consider some outcomes unusual and use that information to make tentative conclusions.

- If students are struggling unduly with questions 3–5, consider giving them loaded dice. (In a loaded dice, the probability of obtaining a particular digit is not necessarily 1/6. Simple instructions for creating loaded dice are available on the internet, or they may be purchased.) After students roll a few times, they may begin to suspect that the dice are loaded. Allow them to continue until they are convinced.

- Question 6: This is a critical question for students to consider. If there are more than 1/3 matches, there are two possible reasons:
  - Reason 1: There is some truth to the theory, and the data reflect this.
  - Reason 2: Everyone was just guessing, and the class just happened by chance to have more than 1/3 of students pick the correct sign.
  - Note: It is also possible that some students may have knowledge about the relationship between personalities traits and zodiac signs.

- The key point here is for students to consider the “it-was-just-luck” explanation. If an individual group is struggling, try to explain by using the legal trial analogy:
  - “If someone has evidence against him or her, does that guarantee he or she is guilty? In this case, what alternative explanation could the ‘defense attorney’ give for the higher number of matches than expected by chance?”

- This is the last question of the group discussion. Make sure that students have thought hard about these questions before you bring them back together for group discussion. Before moving to question 7, review and discuss the groups’ answers, not insisting on specific answers but encouraging them to consider one another’s arguments.

Question 7

- Let students discuss this question in groups, potentially developing very different strategies that can be evaluated as a class. Make sure they are focusing on the legal trial analogy from before (What will the
“prosecutor” do in response to the argument that it was just “luck of the draw”?). The following are some possible prompts:

- “How do you rule out chance? To do that, you need to know what chance looks like. So how do you get an idea for what chance looks like here? How do you create a situation where you know that all that is going on is chance?” (You assume the outcome is chance until convinced otherwise.)

- Bring up the coin-flipping or loaded-dice analogy again (previous page above bullet for question 6):
  - “How did you settle on that cutoff for the number of heads in a row that you could get by chance? How could you check that intuition? How can you determine what outcomes are unlikely to occur just by chance?”

- Have different groups share and critique their strategies. Keep asking what they will learn from their strategy.

- Preview Assignment 1.C models a strategy that could be used for cases with three options.

Wrap-up/transition

(4 minutes)

Wrap-up

- The overall goal is to give students a sense of what a statistical analysis is about. You can talk about the data collection plan and whether it appears sound.
  - For example, are the responses by this class representative of a larger population? Is asking someone to pick one of the three descriptions a good way to answer this research question? Did everyone follow the instructions?

- The larger focus here is actually on the inferential process, which allows you to make generalizations about a claim based on results from a sample. Will you be able to say something about the sign theory in general based on the class results, beyond simply the class results? You know a proportion larger than 1/3 is evidence, but is it convincing enough evidence that students did not just happen to guess correctly more often than expected? You can collect rather easily new information where you know the outcomes you are getting are purely by chance. This allows you to decide whether the data are consistent with chance. If you can rule out chance as an explanation for a class result higher than expected, you have stronger evidence to support the theory.

- In the next tasks, you get more specific about how you can decide whether the class data support this claim. The goal here is to expose students to the reasoning process and ideally help them generate the reasoning process themselves.
• If students do not see the big picture, you can return to the coin example. Ask students whether/why they would be surprised for you to toss 10 heads in a row. They should be able to respond, “Because that would not happen by chance.” You can add, “Assuming I was not doing anything funny when tossing the coin,” but it is the same issue—could this result have happened by chance? If not (or highly unlikely), you consider this evidence that something other than chance is at play.

• It is your option to give the matching descriptions at this point. However, it is recommended that you wait until the matches are revealed in Lesson 1, Part C, as it allows the students to fully experience the simulation strategy in Preview Assignment 1.C.

Transition

• Have students refer back to the Objectives for the lesson and check the ones they recognize from the activity. Alternatively, they may check objectives throughout the lesson.
  o “Let’s think about the people in your group. Do you all have the same personality? How might personality affect our working groups?”

Suggested assessment, assignments, and reflections

[If this is the end of a class meeting, assign the homework for all lesson parts covered that day. If this is simply the transition to another portion of the same class, do not make the assignment until the end of class.]

• Give the Preview Assignments, if any, for the lesson activities you plan to complete in the next class meeting.
Prefering for the next class

In the next lesson, you will need to be able to compute a fraction and a proportion, and estimate the center of a dotplot.

Modeling using envelopes

In the first class meeting, you looked at the relationship between birth dates and personality traits. You were given a choice of three descriptions that might match your astrological sign. In the next class meeting, you will continue to explore this question based on the results obtained by you and your classmates. We can model this activity in the following way:

Consider a statistics class that has 24 students. Each student is given three envelopes and told to pick one. It is important to note that every student has envelopes that are identical to the envelopes held by the other students.

| Envelope 1 | Envelope 2 | Envelope 3 |

1) Each student in the class chooses an envelope and opens it to read the card inside. The cards chosen by six of the students said “Match” while the cards chosen by the remaining students said “No Match.”

Part A: How many students chose cards that said “Match”?

Part B: How many students were in the class?

Part C: What fraction of the students chose cards that said “Match”? 

Part D: What proportion of the students chose cards that said “Match”? 

To determine the proportion of students who selected a match, convert the fraction into a decimal by dividing the number of students with matches by the total number of students. You may choose to use a calculator if you wish. Note that the proportion should be between 0 and 1 (including 0 and 1 as possibilities) since there can never be more matches than there are total students.
2) Each student returns the card to the envelope, shuffles the envelopes, and then chooses a new card. The cards chosen by twelve of the students said “Match” while the cards chosen by the remaining students said “No Match.”

Part A: How many students chose cards that said “Match”?

Part B: How many students were in the class?

Part C: What fraction of the students chose cards that said “Match”?

Part D: What proportion of the students chose cards that said “Match”?

You just completed two “trials.” In this case, a trial consists of every student selecting an envelope and looking at the enclosed card. “Match” indicates that the student chose a card that indicated the correct description for their astrological sign, so randomly selecting one of three cards is similar to saying that birth dates are unrelated to three personality descriptions.

3) Plot the results of your two trials on the dotplot given.
4) Suppose your class completed 100 trials of the proportion of students who selected “Match.” The dotplot of the 100 trials is shown. Estimate the value of the proportion that the trials seem to be clustering around.

![Dotplot of 100 trials showing the proportion of matches]

The trials seem to be clustered around the proportion ________. Can you think of an explanation for this?

5) Which of the following do you think is the most likely description of the three envelopes that each student has? Explain.
   a) 2 “Match” and 1 “No Match”?
   b) 2 “No Match” and 1 “Match”?
   c) There is insufficient information to determine.

**Modeling using cards**

In the first class meeting, you looked at the relationship between birth dates and personality traits. You were given a choice of three descriptions that might match your astrological sign.

In the next class meeting, you will continue to explore this question, based on the results obtained by you and your classmates. We can model this activity in the following
way: Consider three cards below. You can think of these cards as representations of the three lists of personalities given in your first class meeting.

<table>
<thead>
<tr>
<th>Card 1</th>
<th>Card 2</th>
<th>Card 3</th>
</tr>
</thead>
</table>

If you roll a 1 or a 2, you pick Card 1.
If you roll a 3 or a 4, you pick Card 2.
If you roll a 5 or a 6, you pick Card 3.

6) Suppose you give a die to 10 students, who each roll the die once and record the number rolled. Students then pick the corresponding card according to the rules above. Complete the table by rolling a die 10 times, simulating your classmates rolling the die. Each roll can represent a student picking a list of personalities at random. The first roll is done for you.

<table>
<thead>
<tr>
<th>Students</th>
<th>Roll of a die</th>
<th>Card picked</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student 1</td>
<td>5</td>
<td>Card 3</td>
</tr>
<tr>
<td>Student 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Student 3</td>
<td></td>
<td></td>
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<tr>
<td>Student 4</td>
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<td>Student 5</td>
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<td>Student 6</td>
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<td>Student 7</td>
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<td>Student 8</td>
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<td>Student 9</td>
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<tr>
<td>Student 10</td>
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</tr>
</tbody>
</table>

Part A: How many times was Card 1 picked?
Part B: How many times did you roll the die?

Part C: What fraction of the rolls resulted in picking Card 1?

Part D: What proportion of the rolls resulted in picking Card 1?

Part E: Assume it was revealed that Card 1 has the list of personalities that match the zodiac signs, according to astrologists, and assume each student is rolling a die, what proportion of students picked the correct “Match”? 

To determine the proportion of times that Card 1 was picked, convert the fraction into a decimal by dividing the number of times Card 1 was picked by the total number of times the die was rolled. You may choose to use a calculator if you wish. Note that the proportion should be between 0 and 1 (including 0 and 1 as possibilities) since there can never be more number of times a card is picked than there are total rolls.

7) Suppose you give the die again to the 10 students and each rolls it one more time. They record the number they get and pick the corresponding card. The table below shows the results of this second “trial.”

<table>
<thead>
<tr>
<th>Students</th>
<th>Roll of a die</th>
<th>Card picked</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student 1</td>
<td>2</td>
<td>Card 1</td>
</tr>
<tr>
<td>Student 2</td>
<td>6</td>
<td>Card 3</td>
</tr>
<tr>
<td>Student 3</td>
<td>4</td>
<td>Card 2</td>
</tr>
<tr>
<td>Student 4</td>
<td>4</td>
<td>Card 2</td>
</tr>
<tr>
<td>Student 5</td>
<td>5</td>
<td>Card 3</td>
</tr>
<tr>
<td>Students</td>
<td>Roll of a die</td>
<td>Card picked</td>
</tr>
<tr>
<td>------------</td>
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<td>-------------</td>
</tr>
<tr>
<td>Student 6</td>
<td>1</td>
<td>Card 1</td>
</tr>
<tr>
<td>Student 7</td>
<td>5</td>
<td>Card 3</td>
</tr>
<tr>
<td>Student 8</td>
<td>5</td>
<td>Card 3</td>
</tr>
<tr>
<td>Student 9</td>
<td>3</td>
<td>Card 2</td>
</tr>
<tr>
<td>Student 10</td>
<td>4</td>
<td>Card 2</td>
</tr>
</tbody>
</table>

Part A: How many times was Card 1 picked?

Part B: How many times the die was rolled?

Part C: What fraction of the rolls resulted in picking Card 1?

Part D: What proportion of the rolls resulted in picking Card 1?

Part E: Assume it was revealed that Card 1 has the list of personalities that match the zodiac sign according to the astrologist and assume each student is rolling a die, what proportion of students picked the correct “Match”? 
You just completed two “trials.” In this case, a trial consists of every student selecting a card. “Match” indicates that the student chose a card (Card 1) that indicated the correct description for their astrological sign, so randomly selecting one of three cards is similar to saying that birth dates are unrelated to three personality descriptions.

8) Plot the results of your two trials on the dotplot given.

9) Suppose your class completed 100 trials of the proportion of students who selected “Match.” The dotplot of the 100 trials follows. Estimate the value of the proportion that the trials seem to be clustering around.

The trials seem to be clustered around the proportion ________.
Can you think of an explanation for this clustering?
Monitoring your readiness

10) To effectively plan and use your time wisely, it helps to think about what you know and do not know. For each of the following, rate how confident you are that you can successfully do that skill. Use the following descriptions to rate yourself:

5—I am extremely confident I can do this task.
4—I am somewhat confident I can do this task.
3—I am not sure how confident I am.
2—I am not very confident I can do this task.
1—I am definitely not confident I can do this task.

Skills needed for Lesson 1, Part C

<table>
<thead>
<tr>
<th>Skill or concept: I can . . .</th>
<th>Questions used for understanding</th>
<th>Rating from 1 to 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compute a fraction and a proportion.</td>
<td>1–2, 6–7</td>
<td></td>
</tr>
<tr>
<td>Estimate the center of a dotplot.</td>
<td>4, 9</td>
<td></td>
</tr>
</tbody>
</table>

Now use the ratings to get ready for your next lesson. If your rating is a 3 or below, you should get help with the material before class. Remember, your instructor is going to assume that you are confident with the material and will not take class time to answer questions about it.

Ways to get help:
- See your instructor before class for help.
- Ask your instructor for on-campus resources.
- Set up a study group with classmates so you can help each other.
- Work with a tutor.
In Preview Assignment 1.C, you analyzed the dotplot shown. Jot a few sentences that describe your understanding of what each dot represents and of what the dotplot as a whole represents.

Objectives for the lesson
You will begin to understand:

☐ The role that chance variability plays in the statistical decision-making process.

You will be able to:

☐ Describe a categorical data distribution in terms of category frequencies and relative frequencies.
☐ Evaluate the strength of evidence against a claim about a population proportion.

1) What is the largest proportion value shown? What does this proportion value represent? What is the smallest proportion value?

2) How often do proportions with values larger than 0.5 occur? How often do proportions with values smaller than 0.2 occur?

3) How would you complete the following sentences?

“If students in the class were picking at random, it would be unusual to see a match proportion as large as _______.”

“If the personality traits are not related to the zodiac signs, it would be unusual to see a match proportion of students as large as _______.”

4) How does the number you wrote in question 2 relate to what you see in the dotplot?

5) How large would the match proportion have to be to convince Alex, the skeptical student, that personality characteristics might be related to sign? How is this proportion related to your answer to question 3?
Now let’s look at the actual class data and see whether you think that it provides evidence for a relationship between personality traits and astrological sign.

<table>
<thead>
<tr>
<th>Correct Descriptions</th>
<th></th>
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<th></th>
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</thead>
<tbody>
<tr>
<td>Aries</td>
<td>Choice 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Taurus</td>
<td>Choice 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gemini</td>
<td>Choice 3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cancer</td>
<td>Choice 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Leo</td>
<td>Choice 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Virgo</td>
<td>Choice 3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Libra</td>
<td>Choice 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Scorpio</td>
<td>Choice 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sagittarius</td>
<td>Choice 3</td>
<td></td>
<td></td>
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<tr>
<td>Capricorn</td>
<td>Choice 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Aquarius</td>
<td>Choice 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pisces</td>
<td>Choice 3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6) In Lesson 1, Part A, question 2, from the “Astrology Investigation” table, which choice (Choice 1, 2 or 3) did you pick as the best description according to your sign?

   Choice that is a match for your sign?

   Number of students in the class?

   Number of matches in the class?

   Match proportion for the class?

7) Now it all comes down to this! Does the class match proportion provide convincing evidence that personality characteristics could be related to sign? Why or why not? Use the simulation results in your reasoning.

The tasks you have just completed with the astrology example illustrate a general process that is common to many statistical investigations. One way to describe this process is as a sequence of four steps, as shown in the table below.

**Steps in a Statistical Investigation**

1. Ask a question that can be answered by collecting data.

2. Decide what to measure and then collect data.

3. Summarize and analyze the data.

4. Draw a conclusion and communicate the results.

Keep in mind that this process is iterative. Analyzing the results of one study often leads researchers to consider other research questions and then to conduct additional research. Thinking more carefully about how the data were collected may also lead researchers to consider ways to improve the data collection process.
Overview and student objectives

Overview
In this lesson, students debrief Preview Assignment 1.C to formalize their understanding of simulations as a means of modeling. Students are also introduced to the statistical analysis process, which will be explored further in Lesson 1, Part D. The time allotment for Lesson 1, Parts C and D can be fluid, depending on when you choose to hold certain discussions.

Objectives
Students will begin to understand:
- The role that chance variability plays in the statistical decision-making process.

Students will be able to:
- Describe a categorical data distribution in terms of category frequencies and relative frequencies.
- Evaluate the strength of evidence against a claim about a population proportion.

Suggested resources and preparation

Materials and technology
- Computer, projector, document camera
- Preview Assignment 1.C (Students complete prior to class.)
- Student Pages for Lesson 1, Part C
- Practice Assignment 1.C

Prerequisite assumptions
Students should be able to:
- Compute a proportion given the part of a whole that meets a given criteria.
- Read and interpret a dotplot.
Making connections

This lesson connects forward to the formalization of the statistical analysis process and to other simulation experiences.

Background context

In Preview Assignment 1.C, students experienced a simulation of astrological sign making. The results of the simulation were presented in a dotplot that forms the foundation for Lesson 1, Part C. You may wish to play the assignment problem, illustrating the class card selection, the simulation of 100 card selections, and the resulting dotplot.

Suggested instructional plan

Frame the lesson

(5 minutes)

Student Page

- Reference Preview Assignment 1.C:
  - “In your assignment, each ‘student’ chose a card at random and you computed the proportion of a group of students who match the astrologist descriptions. Explain why knowing this one proportion is not enough to tell you the values of the proportion of matches that you would expect to observe if people are truly picking at random.”

  [Answer: This proportion just tells you one value, but what you really need to see is a pattern of values. This outcome could have been common or unusual. You want to look at many outcomes to be able to quantify how common/unusual different values are.]

- Point to the display of the dotplot from the assignment, shown at the beginning of Lesson 1, Part C. Ask students to answer the prompt at the top of their page.

- Debrief student responses to the question.

- Transition to the lesson activities by briefly discussing the Objectives for the lesson.

Lesson activities

(15 minutes)

Small Groups

- Questions 1–5

- Introduce the students to the concept of using simulation to generate data to test whether an outcome is surprising, referring to the simulation from the homework as an example. (For more on simulation, see Teaching with Data Simulations in the Pedagogy in...
Suggested Instructor Notes 1.C, Statistical investigations

Action section of the SERC Portal for Educators:

- Ask students how many trials they think would be necessary to make a sufficient argument that something other than chance is at work. Would 10 trials be sufficient? 100?
- Use this questioning to transition into a discussion about the importance of not using the results from just one class as evidence for or against a claim or hypothesis.

Circulate through the room to observe student responses to questions 1–5.

- Question 3: Note that if the personality traits are not related to the zodiac signs, it means students are picking the lists at random and results look like the ones in the dotplot. The answer is the same as the one before. It would be unusual to see a match proportion as large as 0.5 or 0.6.
- Question 4: Note that class results vary, but they should be out in the “tail” of the distribution generated in the applet, such as “above 0.5.”
- Discussion related to question 5: Although 0.4 is above 0.333, the dotplot from the Preview Assignment shows that it is not extremely far above. It does not appear to be unusual to get 40% correct matches even when you know all the selections are made completely at random, with no correspondence between the astrological sign and personality characteristics. From the dotplot, students should get a sense that 0.4 is closer to the middle of the distribution rather than way out in the tail.
- Some students may go on to count how often matches occur above a proportion of 0.45 or above. For example, you may get 0.45 or more about 16% of the time. You may not want to take students too far down this path right now as they will want to know why you look at “or more” and where the cutoff is. Why is 16% of the time not unusual? What would be unusual?
- If you are around 0.45 or 0.5 or higher, this is a very unusual outcome when the personality characteristics are not related to sign. Class results this large give evidence that students did better than expected just by chance; such results could suggest that there is something to the sign theory.

Questions 5 and 6

- Students need to compare the class results to the dotplot generated by the simulation and see whether the class results are in the tail of the distribution. If the class proportion is in the tail, this outcome provides evidence that personality characteristics might be related to sign or at least evidence that random chance alone is not a sufficient counter argument.
Wrap-up/transition
(5 minutes)

Wrap-up

- Formal discussion of the steps in a statistical investigation can be done here or in the next lesson activity. However, be sure at least to informally discuss the overall process that students have experienced here—state a theory, collect data to test the theory, and consider what kind of evidence provides support for this theory. For example, in this case, students evaluated two possible explanations:
  - 1) The theory is correct.
  - 2) The theory is not correct, in which case the observed result is due to chance variability.
- To conclude that there is evidence in support of the theory, they needed to rule out chance as a plausible explanation for what was observed. First, students generated results, assuming that the theory was false and what they were seeing was only due to chance variability; they then compared the observed results to see whether they can be considered consistent with chance.
- Have students refer back to the Objectives for the lesson and check the ones they recognize from the activity. Alternatively, they may check objectives throughout the lesson.

Suggested assessment, assignments, and reflections

[If this is the end of a class meeting, assign the homework for all lesson parts covered that day. If this is simply the transition to another portion of the same class, do not make the assignment until the end of class.]

- Give Practice Assignment 1.C.
- Give the Preview Assignments, if any, for the lesson activities you plan to complete in the next class meeting.
1) Suppose the investigation had given each student a choice of four personality descriptions for each astrological sign. If there is no relationship between personality characteristics and sign, about what fraction of the students in the class do you expect to pick the description that astrologists say matched their sign?

2) One of the two dotplots shown below was constructed after asking 40 students to pick one of four descriptions at random (using index cards) and then computing the proportion that chose the description matching their sign just by chance. This process was repeated a large number of times to generate the data used to construct the dotplot. Which dotplot do you think is the one that was constructed in this way? Circle your choice below.

(Note: You will explain your reasoning in the next question.)

Dotplot 1 OR Dotplot 2

3) Explain your reasoning for your choice of dotplot in the previous question.
4) Suppose each of these 40 students then made their choices at random from the list of four personality types. What proportion of the 40 students need to match correctly to provide convincing evidence that there is a connection between sign and personality type?

5) Suppose the investigation had given each student a choice of two personality descriptions, instead of three, for each astrological sign. If there is no relationship between personality characteristics and sign, about what fraction of the students in the class do you expect to pick the description that astrologists say matched their sign?

6) One of the two dotplots shown below was constructed after asking 40 students to pick one of two descriptions at random (using a balanced coin; heads for description one and tails for description two) and then computing the proportion that chose the description matching their sign just by chance. This process was repeated a large number of times to generate the data used to construct the dotplot. Which dotplot do you think is the one that was constructed in this way? Circle your choice below.

(Note: You will explain your reasoning in the next question.)

<table>
<thead>
<tr>
<th>Dotplot 1</th>
<th>OR</th>
<th>Dotplot 2</th>
</tr>
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</table>

![Dotplot 1](image1.png)

![Dotplot 2](image2.png)
7) Explain your reasoning for your choice of dotplot in the previous question.

8) Suppose each of these 40 students then made their choices at random from the list of two personality types. What proportion of the 40 students need to match correctly to provide convincing evidence that there is a connection between sign and personality type?