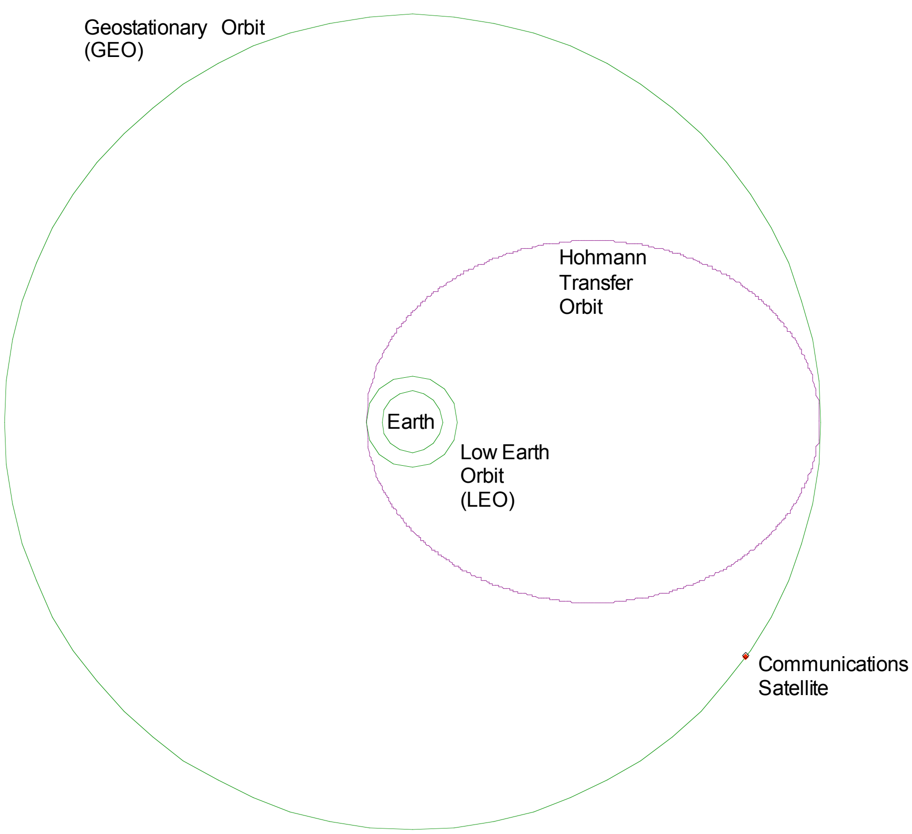
TASK 7.4.6: COMMUNICATION SATELLITE MISSION DESIGN

Solutions

The aerospace company for which you work is launching a new communications satellite for satellite TV. The satellite must be positioned so that the time it takes to complete one orbit is exactly equal to the time it takes the Earth to complete one orbit. This will make the satellite appear to stand still when viewed from Earth, making it possible to aim the customer's satellite TV dish at a fixed location in the sky.

The satellite will first be launched into a low orbit around the Earth, as shown below. Once this orbit is established and the satellite is tested, it will then be sent into its final orbit at a greater distance from the Earth. The satellite will follow an elliptical path from a low Earth orbit (LEO) to its final geostationary orbit (GEO). This path is called a Hohmann transfer orbit. It is an ellipse that is tangential to both circular orbits and has a focus at the center of the Earth. Your task is to find the equation for the ellipse that describes this transfer orbit.



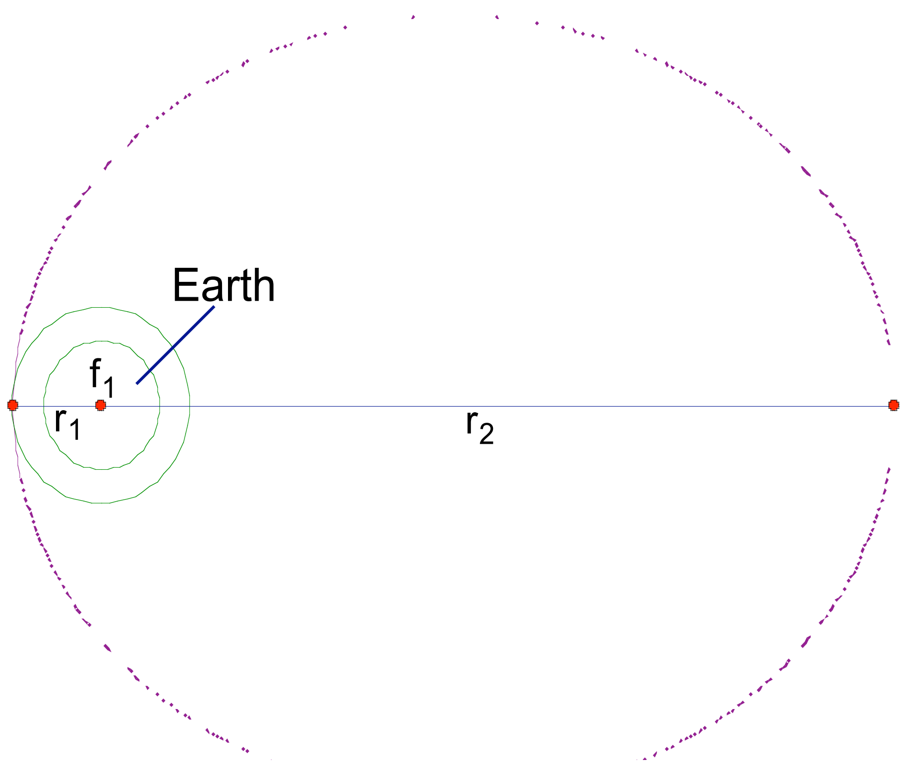
The first thing that you do is establish the origin of your coordinate system as the center of the Earth. Next, you recall that the radius of the Earth is 6378 km. You have been told that the initial orbit (LEO) will be at an altitude of 300 km above the Earth. You also know that the period required for one rotation of the Earth is 23 hours 56 minutes. Now, you dig in an old textbook and find that the equation that relates the period of an orbit to the radius of the orbit is:

$$P = 2\pi\sqrt{\frac{r^3}{398601}}$$

Where: P = period of the orbit in seconds
r = radius of the orbit in km

Given this information, you now have everything you need to find the equation of the transfer orbit. Good luck! And…don't forget to show all of your calculations. You will have to justify your solution to the rest of the launch team.

Students may have difficulty in knowing where to start. Begin by leading them to make a labeled drawing as follows:



The solution process is described below. Ask leading questions to help students discover for themselves the steps in the solution.

Ask, ***"Do we know r_1?"*** *No.* ***"How can we find r_1 from the information given in the problem?"***

We know the radius of the Earth, 6378 km, and we know the altitude of the LEO, 300 km; therefore, we know r_1. $r_1 = 6378 \text{ km} + 300 \text{ km} = 6678 \text{ km}$.
Ask, ***"Do we know r_2?"*** *No.* ***"How can we find r_2 from the information given in the problem?"***
Radius r_2 is the radius of the GEO. We do not know this value, but we do know the period of the GEO, therefore, we can use the equation $P = 2\pi\sqrt{\frac{r^3}{398601}}$ to find r_2.
We must solve this equation for r and then substitute to find the value of r_2:

$$P = 2\pi\sqrt{\frac{r^3}{398601}}$$

$$\frac{P}{2\pi} = \sqrt{\frac{r^3}{398601}}$$

$$\left(\frac{P}{2\pi}\right)^2 = \left(\sqrt{\frac{r^3}{398601}}\right)^2$$

$$\left(\frac{P}{2\pi}\right)^2 = \frac{r^3}{398601}$$

$$398601\left(\frac{P}{2\pi}\right)^2 = r^3$$

$$r = \sqrt[3]{398601\left(\frac{P}{2\pi}\right)^2}$$

P must be in seconds; therefore, we must convert the period of 23 hours 56 min to seconds:

$$23 \text{ hrs} \cdot \frac{60 \text{ min}}{\text{hr}} \cdot \frac{60 \text{ sec}}{\text{min}} = 82800 \text{ secs}$$

$$56 \text{ min} \cdot \frac{60 \text{ sec}}{\text{min}} = 3360 \text{ secs}$$

$$82800 \text{ secs} + 3360 \text{ secs} = 86160 \text{ secs}$$

Substituting, we find

$$r_2 = \sqrt[3]{398601\left(\frac{86160}{2\pi}\right)^2} = 42163 \text{ km}$$

Ask, "***Now that we know both r_1 and r_2, how does this help us find the equation of the ellipse?"*** Give them some think time. If they need a hint, just say the letters, "***a, b, c***."

Looking back at our drawing, we know r_1 and r_2. Using what we know about an ellipse, we recognize that this is 2a, the length of the major axis. We can now find a:

2a = 6,678 km + 42,163 km = 48,841 km

$$a = \frac{48,841}{2} \text{ km} = 24,420.5 \text{ km}$$

Next, knowing that the center of the Earth is a focus of the ellipse, we can find c:

$2c = 2a - 2r_1$ = 48841 km – 2(6678 km) = 35,485 km

$$c = \frac{35485}{2} \text{ km} = 17,742.5 \text{ km}$$

Knowing a and c, we can now find b, since $a^2 = b^2 + c^2$. Rearranging, we can solve for b^2:

$$b^2 = a^2 - c^2$$

$$b = \sqrt{(24420.5)^2 - (17742.5)^2} = 16,680 \text{ km}$$

The final step is to find the center of the transfer ellipse. The center of the Earth is a focus of the ellipse. This is also the origin of our coordinate system. The distance from a focus to the center of an ellipse is c; therefore, the coordinates of the center of the ellipse are (c, 0). Knowing this, we can now use the standard equation of an ellipse, where (h, k) is the center of the ellipse:

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

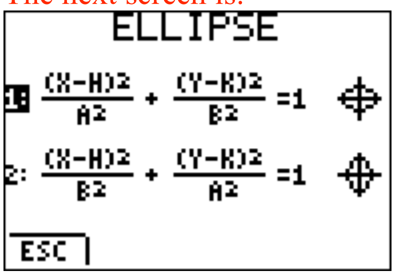
Substituting, we find:

$$\frac{(x-17742.5)^2}{(24420.5)^2} + \frac{y^2}{(16680)^2} = 1$$

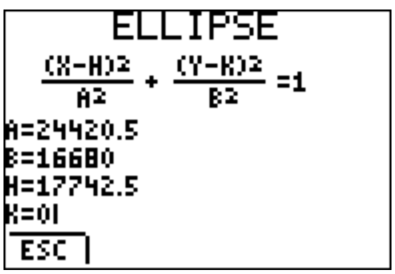
The last step is to verify our solution. We will use the Conics APP on the TI-83+ to check our result. To start the app, press the blue **APPS** key. Choose **5:Conics**. The following screen will appear:

Choose **2: ELLIPSE**

The next screen is:



Choose **1** and enter the appropriate value for each variable.



Enter:

A = 24420.5
B = 16680
H = 17742.5
K = 0

Press **GRAPH**.

Use **TRACE** to verify that the vertices on the major axis are (-6678,0) and (42163,0). This corresponds to our calculated values of r_1 = 6,678 km and r_2 = 42,163 km; therefore, our equation describes an ellipse that connects exactly between the LEO and GEO. We successfully found the equation for the Hohmann transfer orbit. Mission accomplished!

The link below will take you to NASA's JTrack 3D satellite tracking page which allows you to track satellites in real-time. If you have either a video projector or a large screen TV connected to a computer, this page is a must to verify the calculations of this student task. Once the browser window opens, the tracking window will open on top of it in the upper left-hand corner. Click to maximize the tracking window. Click on Options and select Update Rate as Second. This will update the position every second. Click Options again. Set Timing to Real-time. This will cause the program to calculate and show the real-time position of the satellite. Students can actually observe the satellites moving on the display. You can select any satellite by double-clicking on it. Select one that is in the large circular orbit. These are all geosynchronous. Click on View and select Satellite Position. Another window will open that will display the current position, altitude, velocity, period, and inclination of the selected satellite. Have students compare the actual NASA data with their calculated values.

http://science.nasa.gov/Realtime/JTrack/3d/JTrack3D.html
Visit the NASA Marshall Spaceflight Center website below for more information on the mathematics of orbits, Hohmann transfers, and geosynchronous satellites:

http://liftoff.msfc.nasa.gov/toc.asp?s=Orbital%20Mechanics

Suggested Reference

Lee, Wayne. (1995). *To Rise From Earth: An Easy-to-Understand Guide to Spaceflight.* New York: Facts on File.

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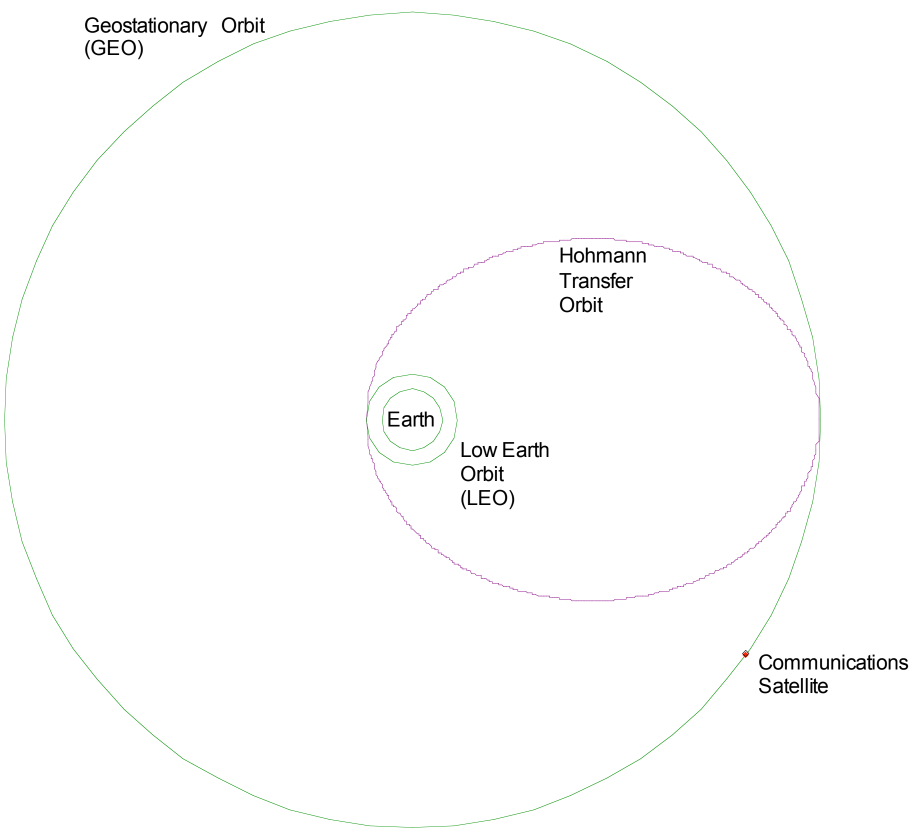

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