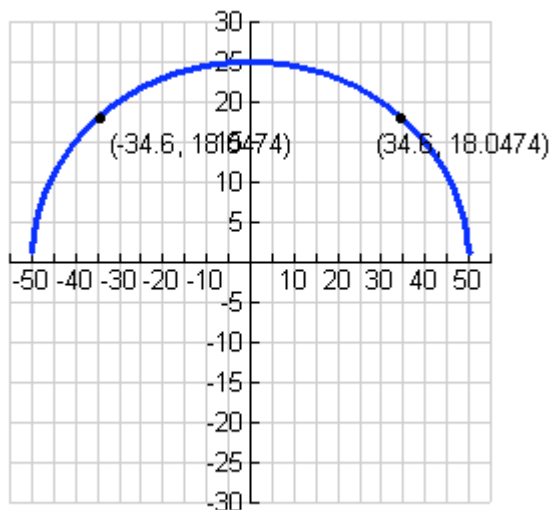


TASK 7.4.3: AN ARCHING DILEMMA**Solutions**

The arch of a tunnel through Green Mountain is a semi-ellipse 100 ft. wide and 25 ft. high. If the overhead clearance must be at least 18 ft., how many lanes of highway 10 ft. wide can be built through the tunnel?



The tunnel might be graphed as half of an ellipse with its center at the origin, major axis 100 units long, and semi-minor axis 50 units long.

Therefore $a = 50$ and $b = 25$ and the equation of the ellipse is $\frac{x^2}{50^2} + \frac{y^2}{25^2} = 1$.

Solve this equation for y : $y = \pm 25 \sqrt{\left(1 - \frac{x^2}{50^2}\right)}$ and graph only the positive solution.

Trace on the graph to see that $y > 18$, where $-34.6 \leq x \leq 34.6$.

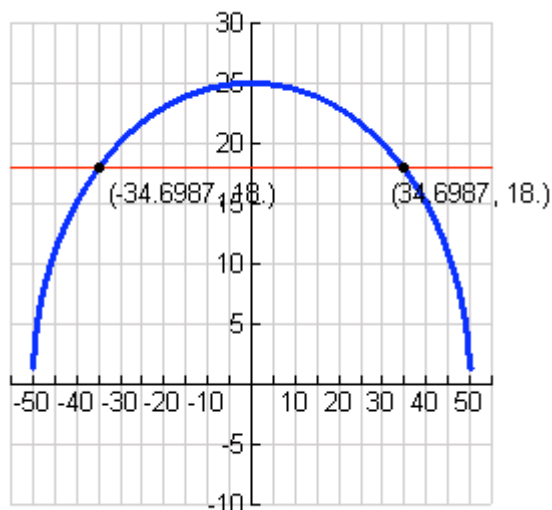
The distance between -34.6 and 34.6 is 69.2 .

Therefore, 6 lanes can be built.

One might explore a more accurate answer to see if in fact 7 lanes could be built.

To do this, solve the system

$$y = 25 \sqrt{\left(1 - \frac{x^2}{50^2}\right)}, \quad y \geq 18$$



$$18 = 25 \sqrt{1 - \frac{x^2}{50^2}}$$

$$\frac{18}{25} = \sqrt{1 - \frac{x^2}{50^2}}$$

$$\left(\frac{18}{25}\right)^2 = 1 - \frac{x^2}{50^2}$$

$$\frac{x^2}{50^2} = 1 - \left(\frac{18}{25}\right)^2$$

$$x^2 = 50^2 - \left(\frac{18}{25}\right)^2 * 50^2$$

$$x^2 = 1204$$

$$x = \pm\sqrt{1204} \approx 34.6987$$

A graphical solution yields $-34.6987 \leq x \leq 34.6987$ or a distance of 69.3974 ft., which means the answer is still only 6 lanes.

An exact algebraic solution still does not change the answer.

Only six lanes having the necessary clearance will fit in the tunnel. Note: This problem gives a golden opportunity to discuss when and how to round solutions.

Teaching notes

Let the participants work on the problem in pairs. Allow time for them to read and set up the problem before discussing as a group. Answer questions as they arise.

As an after discussion, point out that students may not automatically see the relationship between horizontal distance and x-coordinates and vertical clearance and y-coordinates.

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