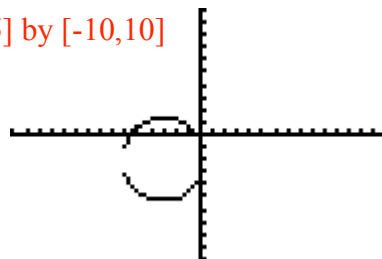


TASK 7.3.1: GRAPHING CONICS $AX^2 + CY^2 + DX + EY + F = 0$ **Solutions**

- For each equation, use the program *CONICSTU* to graph the conic.
- After executing the program, you may need to adjust the window to see a complete graph.
- Sketch the graph. Show the graphing window.
- What type of conic is formed when the equation is graphed?
- Which constants affect the type of conic that is graphed? **The constants that seem to affect the graph most are A and C . D , E , F affect the elongation of an ellipse and whether parabolas and hyperbolas open up, down, left, right.**
- What generalizations do we usually make concerning these constants?
 - If $A = C$, then a circle is graphed.**
 - If $A \neq C$, $A > 0$ and $C > 0$, then an ellipse is graphed.**
 - If $A \neq C$, $A < 0$ and $C > 0$, then a hyperbola is graphed.**
 - If $A \neq C$, $A > 0$ and $C < 0$, then a hyperbola is graphed.**
 - If A or C is zero and the other is not, then a parabola is graphed.**

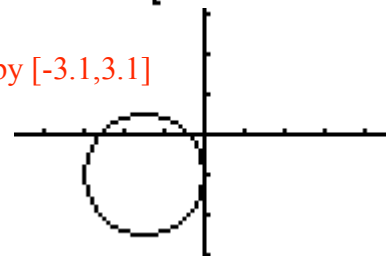
1. $x^2 + y^2 + 6x + 4y + 2 = 0$ Circle

[-15,15] by [-10,10]



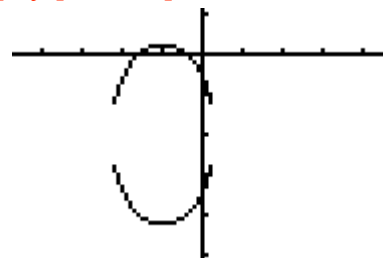
2. $2x^2 + 2y^2 + 6x + 4y + 2 = 0$ Circle

[-4.7,4.7] by [-3.1,3.1]



3. $3x^2 + y^2 + 6x + 4y + 2 = 0$ Ellipse

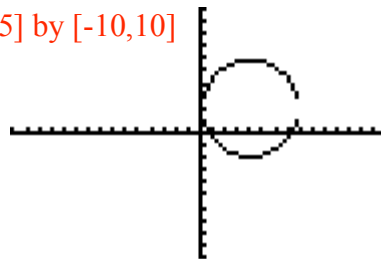
[-4.7,4.7] by [-5.1,1.1]



Algebra II: Strand 7. Conic Sections; Topic 3. Graphing Conics; Task 7.3.1

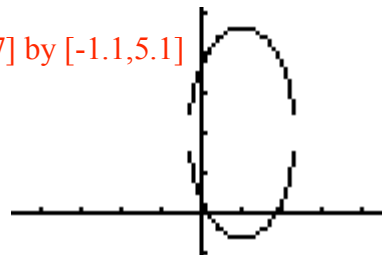
4. $x^2 + y^2 - 8x - 4y + 5 = 0$ Circle

$[-15,15]$ by $[-10,10]$



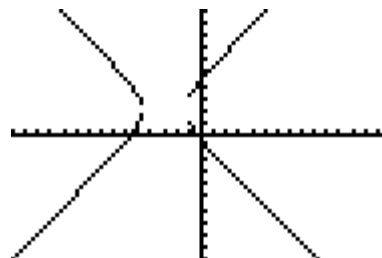
5. $4x^2 + y^2 - 8x - 4y + 1 = 0$ Ellipse

$[-4.7,4.7]$ by $[-1.1,5.1]$



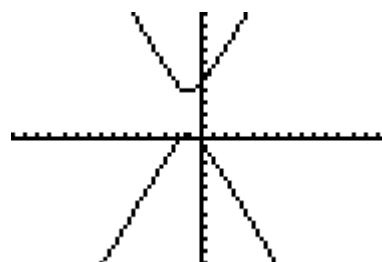
6. $x^2 - y^2 + 6x + 4y + 2 = 0$ Hyperbola

$[-15,15]$ by $[-10,10]$



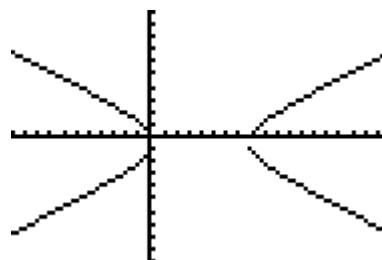
7. $3x^2 - y^2 + 6x + 4y + 2 = 0$ Hyperbola

$[-15,15]$ by $[-10,10]$



8. $x^2 - 4y^2 - 8x - 4y + 1 = 0$ Hyperbola

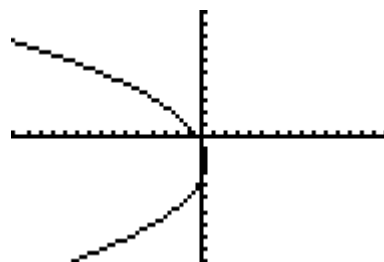
$[-11,19]$ by $[-10,10]$



Algebra II: Strand 7. Conic Sections; Topic 3. Graphing Conics; Task 7.3.1

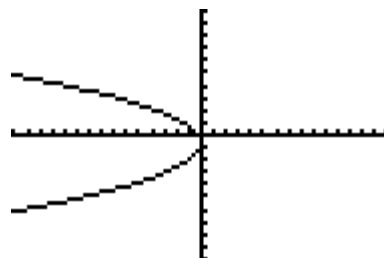
9. $y^2 + 6x + 4y + 2 = 0$ Parabola

$[-15,15]$ by $[-10,10]$



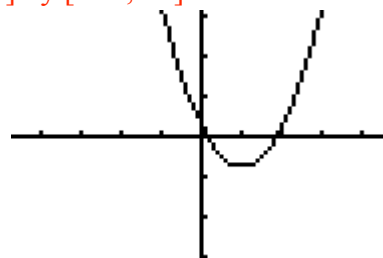
10. $3y^2 + 6x + 4y + 2 = 0$ Parabola

$[-15,15]$ by $[-10,10]$



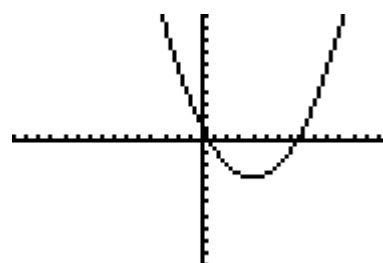
11. $4x^2 - 8x - 4y + 1 = 0$ Parabola

$[-4.7,4.7]$ by $[-3.1,3.1]$



12. $x^2 - 8x - 4y + 4 = 0$ Parabola

$[-15,15]$ by $[-10,10]$



Algebra II: Strand 7. Conic Sections; Topic 3. Graphing Conics; Task 7.3.1

Use the method of completing the square to change the following equations to standard form. Discuss how the graphs you drew might help you to do this. Using the standard form equation, describe the graph.

<p>4. $x^2 + y^2 - 8x - 4y + 5 = 0$</p> $x^2 + y^2 - 8x - 4y + 5 = 0$ $x^2 - 8x + y^2 - 4y = 0 - 5$ $x^2 - 8x + 16 + y^2 - 4y + 4 = 0 - 5 + 16 + 4$ $(x - 4)^2 + (y - 2)^2 = 15$ <p>This is a circle with center at (4,2) and radius of $\sqrt{15}$.</p> <p>My graph had a center of (4,2) which helps me to know what the binomial squares should be.</p>	<p>7. $3x^2 - y^2 + 6x + 4y + 2 = 0$</p> $3x^2 - y^2 + 6x + 4y + 2 = 0$ $3x^2 + 6x - y^2 + 4y = 0 - 2$ $3(x^2 + 2x) - (y^2 - 4y) = -2$ $3(x^2 + 2x + 1) - (y^2 - 4y + 4) = -2 + 3(1) + -1(4)$ $3(x + 1)^2 - (y - 2)^2 = -3$ $\frac{(y - 2)^2}{3} - \frac{(x + 1)^2}{1} = 1$ <p>This is a hyperbola with center at (-1,2) which opens up and down. The vertices are (-1, 2+$\sqrt{3}$) and (-1, 2-$\sqrt{3}$).</p> <p>My graph had a center of (-1,2) which helps me to know what the binomial squares should be.</p>
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Math notes

Be sure to note that we graph relations on the graphing calculator by graphing two functions that generate the same set of points.

Teaching notes

Introduce this activity by using Transparency #1.

Cover the bottom of the transparency so that only the conic to be graphed is visible.

Ask participants what they need to do to be able to graph this equation.

When someone responds that they need to solve the equation for y, work through the rest of the transparency together. Then use your calculator to display the graph.

If they mention *completing the square* and changing to standard form, honor that it can be done this way and ask a volunteer to prepare a transparency showing the necessary work. Save the transparency for later in the activity.

Look at Task 7.3.1 Graphing Conics $Ax^2 + Cy^2 + Dx + Ey + F = 0$ and discuss graphing these equations, emphasizing that the work would be the same each time. State that when we do the same task repetitively, mathematicians often create a formula for doing the work.

Algebra II: Strand 7. Conic Sections; Topic 3. Graphing Conics; Task 7.3.1

Ask participants if they can generalize their work to graph

$$Ax^2 + Cy^2 + Dx + Ey + F = 0$$

Work through Transparency #2 together to illustrate this.

Display a copy of the program **CONICSTU** and discuss what it does. Use it to graph the equation on Transparency #1.

Allow participants to work on the first page of Task 7.3.1 in groups of 3 or 4, dividing up the work so that they can look at all 12 graphs in a reasonable amount of time. They should discuss all of the questions on this page.

When they have finished, hold a class discussion regarding the generalizations they made, asking a volunteer to record them on chart paper or a transparency.

Now discuss the method of *completing the square* using the equation from Transparency #1. If your participants mentioned it in the earlier discussion, then you have the work already recorded on a transparency. If not, do the work together as a group.

Complete page 2 of Task 7.3.1.

Technology notes

Due to character transmission issues, the **CONICSTU** program might not run properly if it is copied and pasted (and then sent to your TI device) electronically. If the program does not run, there are two things that need to be fixed on the calculator. Edit the program and make these changes:

1. Insert negative signs where appropriate. In transmission, negative signs were changed to subtraction signs
2. Insert function variables where appropriate (Y1, Y2, Y3, Y4). In transmission, these variables were separated into two distinct characters. Go to VARS then Y-VARS then FUNCTION and insert the appropriate variable

TI Program: CONICSTU

```
PlotsOff
FnOff
AxesOff
Text(8,0,"TO GRAPH THE ")
Text(18,0,"GENERAL CONIC")
Text(28,0,"AX^2+CY^2+DX+EY+F=0")
Text(38,0,"ENTER THE COEFFICIENTS")
Text(55,68,"[ENTER]")
Pause
FnOff
PlotsOff
AxesOn
Simul
```

Algebra II: Strand 7. Conic Sections; Topic 3. Graphing Conics; Task 7.3.1

```

Func
-15→Xmin
15→Xmax
1→Xscl
-10→Ymin
10→Ymax
1→Yscl
Connected
Simul
Disp "ENTER"
Disp "COEFFICIENTS"
Disp "A"
Input A
Disp "C"
Input C
Disp "D"
Input D
Disp "E"
Input E
Disp "F"
Input F
If C=0
Then
"(-AX^2-DX-F)/E"→Y4:End
"√((E)^2-4C(AX^2+DX+F))"→Y1
"(-(E)+Y1)/(2C)"→Y2
"(-(E)-Y1)/(2C)"→Y3
FnOff 1
DispGraph

```

Transparency #1 Graphing Conics

Graph:

$$3x^2 + 4y^2 - 11x + 27y - 50 = 0$$

Use the quadratic formula to solve this equation for y :

$$3x^2 + 4y^2 - 11x + 27y - 50 = 0$$

Rewrite the equation in descending powers of y :

$$4y^2 + 27y + 3x^2 - 11x - 50 = 0$$

The general equation for a quadratic equation in y is:

$$ay^2 + by + c = 0$$

$$a = 4 \quad b = 27 \quad c = 3x^2 - 11x - 50$$

Applying the quadratic formula:

$$y = \frac{-b \pm \sqrt{b^2 - 4(a)(c)}}{2(a)}$$

$$y = \frac{-27 \pm \sqrt{27^2 - 4(4)(3x^2 - 11x - 50)}}{2(4)}$$

$$y = \frac{-27 + \sqrt{27^2 - 4(4)(3x^2 - 11x - 50)}}{2(4)}$$

and

$$y = \frac{-27 - \sqrt{27^2 - 4(4)(3x^2 - 11x - 50)}}{2(4)}$$

Transparency #2

$$Ax^2 + Cy^2 + Dx + Ey + F = 0$$

Solve this equation for y .

$$Ax^2 + Cy^2 + Dx + Ey + F = 0$$

Write in descending powers of y .

$$Cy^2 + Ey + Ax^2 + Dx + F = 0$$

$$a = C \quad b = E \quad c = Ax^2 + Dx + F$$

$$y = \frac{-E \pm \sqrt{E^2 - 4C(Ax^2 + Dx + F)}}{2C}$$

Transparency: CONICSTU

```

PlotsOff
FnOff
AxesOff
Text(8,0,"TO GRAPH THE ")
Text(18,0,"GENERAL CONIC")
Text(28,0,"AX^2+CY^2+DX+EY+F=0")
Text(38,0,"ENTER THE COEFFICIENTS")
Text(55,68,"[ENTER]")
Pause
FnOff
PlotsOff
AxesOn
Simul
Func
-15→Xmin
15→Xmax
1→Xscl
-10→Ymin
10→Ymax
1→Yscl
Connected
Simul
Disp "ENTER"
Disp "COEFFICIENTS"
Disp "A"
Input A
Disp "C"
Input C
Disp "D"
Input D
Disp "E"
Input E
Disp "F"

```

Input F

If C=0

Then

"(-AX²-DX-F)/E" → Y₄:End

"√((E)²-4C(AX²+DX+F))" → Y₁

"(-(E)+Y₁)/(2C)" → Y₂

"(-(E)-Y₁)/(2C)" → Y₃

FnOff 1

DispGraph

TASK 7.3.1: GRAPHING CONICS $AX^2 + CY^2 + DX + EY + F = 0$

- For each equation, use the program *CONICSTU* to graph the conic.
- After executing the program, you may need to adjust the window to see a complete graph.
- Sketch the graph. Show the graphing window.
- What type of conic is formed when the equation is graphed?
- Which constants affect the type of conic that is graphed? And, how?
- What generalizations do we usually make concerning these constants?

1. $x^2 + y^2 + 6x + 4y + 2 = 0$
2. $2x^2 + 2y^2 + 6x + 4y + 2 = 0$
3. $3x^2 + y^2 + 6x + 4y + 2 = 0$
4. $x^2 + y^2 - 8x - 4y + 5 = 0$
5. $4x^2 + y^2 - 8x - 4y + 1 = 0$
6. $x^2 - y^2 + 6x + 4y + 2 = 0$
7. $3x^2 - y^2 + 6x + 4y + 2 = 0$
8. $x^2 - 4y^2 - 8x - 4y + 1 = 0$
9. $y^2 + 6x + 4y + 2 = 0$
10. $3y^2 + 6x + 4y + 2 = 0$
11. $4x^2 - 8x - 4y + 1 = 0$
12. $x^2 - 8x - 4y + 4 = 0$

Use the method of completing the square to change the following equations to standard form. Discuss how the graphs you drew might help you to do this. Using the standard form equation, describe the graph.

4. $x^2 + y^2 - 8x - 4y + 5 = 0$
7. $3x^2 - y^2 + 6x + 4y + 2 = 0$