TASK 6.2.2: BACTERIA GROWTH

Solutions

E. coli bacteria are very small – 1 micrometer in length. The following discussion about the size of a micrometer might help to put its size in perspective. You may want to use a meter stick during this part of the discussion to help participants visualize how tiny a micrometer is.

1. One micrometer = 10^{-6} meter or $\frac{1}{1,000,000}$ of a meter. A millimeter is *10^{-3} meter or* $\frac{1}{1,000}$ *of a meter. A micrometer, therefore, is* $\frac{1}{1,000}$ *of a millimeter. Another way of looking at this situation is as follows: if 1 micrometer = 10^{-6} meters, then 1 meter = ? micrometers.* *(We want participants as well as students to understand relationships in both directions. Do students understand that if 1 micrometer = 10^{-6} or* $\frac{1}{1,000,000}$ *of a meter, then 1 meter = 10^{6} or 1,000,000 micrometers?)*

2. How many micrometers is a five-foot tall person?

   Several approaches are shared below.

   (1) Using a meter stick, we find that five feet = 60 inches ≈ 152 cm = 1.52 m. Knowing that a five-foot person is approximately 1.52 meters tall and there are 1,000,000 micrometers for each meter, we should expect the person's height to be approximately 1.52 million micrometers.

   (2) Using dimensional analysis, we get:

$$5 \text{ feet} \times \frac{12 \text{ in.}}{1 \text{ ft.}} \times \frac{2.54 \text{ cm}}{1 \text{ in.}} \times \frac{1 \text{ meter}}{100 \text{ cm}} = 1.524 \text{ m}$$

   Because a meter is equal to 1,000,000 micrometers, we can compute a five-foot tall person's height in micrometers.

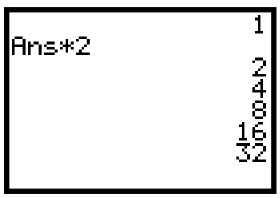
$$1.524 \text{ m} \times \frac{1,000,000 \text{ micrometers}}{1 \text{ meter}} = 1,524,000 \text{ micrometers}$$

   Hence we can conclude that the height of a 5-foot tall person is approximately equal to the length of 1,524,000 E. coli laid end-to-end.

3. Suppose each *E. coli* cell divides once a minute and we start with 1 bacterium.
   a. Build a table of values to model the E. coli growth. Participants should build a table of values like the table below.

Number Minutes	**Cell Division**	**Number *E. coli***
0	o	1
1	o o	2
2	o o o o	4
3	o o o o o o o o	8
4	o o o o o o o o o o o o o o o o	16

   b. Participants use the recursive feature the home screen of a graphing calculator to model the *E. coli* growth.
   To do this, ask participants to analyze the table above and decide what operation can be performed on a value of the number of *E. coli* to give the next value. Now, in the home screen of the calculator, enter the initial number of *E. coli,* 1. Then type the operation (for example +2 or *3 or whatever they decided). When you hit the enter key, the calculator performs the operation on the previous entry. Continue to hit the enter key and see the values for the number of *E. coli*.



   c. Participants create a scatter plot of the data and determine a mathematical model that accurately describes this set of data. Participants may recognize this table of values as one that satisfies $y = 2^x$. Others may choose to use the calculator's curve-of-best-fit feature to determine the mathematical model. The number of *E. coli* is the starting amount times the common quotient raised to the time or the starting amount (1) times quotient, 2, raised to time or $y = 1 \cdot 2^x$.

   A teacher may provide several functions for the students to test in order to find a model. For example, the teacher might provide $y = 2x$, $y = \frac{1}{2}x$, $y = x^2$, $y = x^{\frac{1}{2}}$, $y = 2^x$, $y = \left(\frac{1}{2}\right)^x$ as possible choices and allow the students to experiment with the given functions to determine the best choice.

d. How many cells will we have after 1 hour? How many after 2 hours? Describe how you arrived at your solution. Remind participants about the Table feature or the Calculate Value feature of the calculator to determine how many cells there will be after 1 hour. Ask participants to answer the question and describe the method they used. *After* 1 *hour, there will be approximately* 1.15×10^{18} *or* 1,152,921,505,000,000,000 *E. coli cells. After* 2 *hours, there will be approximately* 1.33×10^{36} *or* 1,329,227,996,000,000,000, 000,000,000,000,000,000 *E. coli cells.*

e. How long will it take 1 E. coli cell to divide so that 1 billion cells are formed? Describe your solution strategy.
This question allows us to ask when does y = 1,000,000,000 or when does 2^x= 1,000,000,000. Use **Instructor Transparency 1** to help participants see how the need for equations follows naturally from the study of functions. The solution is 29.897 minutes or approximately 30 minutes.

Possible Solution Strategies:

(1) Guess and check in the home screen. Guess a value for *x* that would make $2^x = 10^9$.

(2) Use the Calculate Intersection feature of the calculator.

Enter $Y_1 = 2^x$ and $Y_2 = 1,000,000,000$, graph, and have the calculator determine the point of intersection.

(3) Use the Table feature of the calculator. Enter $Y_1 = 2^x$ and scan the Y_1 column of the Table for 1,000,000,000. The corresponding *x*-value should be approximately 30.

(4) Use logarithms to solve $2^x = 1,000,000,000$ or 10^9.

$$\log 2^x = \log 10^9$$

$$x \log 2 = 9 \log 10$$

Because the log 10 is equal to 1, we next get:

$$x \log 2 = 9$$

$$x = \frac{9}{\log 2}$$

$$x = \frac{9}{0.301029957...}$$

$x = 29.89735285...$ or approximately 30 minutes

Math notes
The power of this task is how the exponential equations arise naturally in context.

Teaching notes
Begin a discussion about *E. coli* with the whole group.
What are *E. coli* and where have you heard of them? *E. coli, short for Escherichia coli, is one of the most thoroughly studied of all organisms. Normally, this organism lives in the intestinal tract of humans and aids the digestive system. Only certain types of E. coli cause problems for the human host.*

- *E. coli* are bacteria that live in the human intestinal tract.
- Most strains are harmless and aid digestion.
- An *E. coli* bacterium is rod- shaped and 1 micrometer long.
- The growth of *E. coli* provides a good model for population growth in general.

Allow participants to consider the questions in Task 6.2.2 in groups followed by a whole group discussion of findings.

As students complete questions 1 and 2, bring the class together for a group discussion on how fast the E. coli grow before they work on question 3.

> How fast can *E. coli* grow? *Cells of E. coli reproduce by simply splitting in half, so that one cell forms two "daughter cells." This form of cell division is called* ***fission***. Fission occurs at a rate that depends on the nutrients and conditions (e.g., temperature) that are available. When conditions are ideal, *E. coli* can reproduce very rapidly. On the other hand, in a restrictive environment, one in which the food supply is scarce or conditions are poor in other ways, cell division may slow down or stop altogether.

In part 3d, the following questions should be posed as the leader circulates among the groups:

- Describe in words how the *E. coli* bacteria in this situation grow? *They double every minute.*
- Does the value of the number of *E. coli* increase by repeated addition or repeated multiplication? *Repeated multiplication.*
- What do we call this type of procedure? *Recursion.*
- What kind of model do we get for the bacteria growth? *Exponential.*
- Would we always get this type of model if growth is modeled by repeated multiplication? *Yes.*
- What would happen if we looked at a decaying situation modeled by "repeated division"? *Repeated division is the same thing as repeated multiplication by a fraction between 0 and 1. In other words, repeated division by 2 is the same thing as repeated multiplication*

by $\frac{1}{2}$. We would still have an exponential model, but the function would be decreasing.

For example, $y = \left(\frac{1}{2}\right)^x = 2^{-x}$.

For part 3e, the leader should have participants present on transparencies their various solution strategies. Bring out as many possible solution strategies as possible including (4) in the solution notes above. This is a natural way to include some discussion on logarithms.

Instructor Transparency 1

Questions Can Lead from Functions to Equations

Question:

How long will it take
1 *E. coli* cell to divide
and form 1 billion cells?

Functions:

$y = 2^x$ and $y = 10^9$
(where x = number of minutes elapsed and y = number of cells)

Equation: $2^x = 10^9$

The equation forms naturally from the study of the two functions:

$y_1 = 2^x$ and $y_2 = 10^9 = 1{,}000{,}000{,}000$.

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   If 1 micrometer = 10^{-6} meters, then 1 meter = _______ micrometers.

2. How many micrometers is a five-foot tall person? Explain how you arrived at your solution.

3. Suppose each *E. coli* cell divides once a minute and we start with 1 bacterium.
   a. Build a table of values to model the *E. coli* growth.

Number of minutes	**Cell Division**	**Number of *E. coli***
0	o	1
1	o o	2
2	o o o o	
3		
4		

   b. Use the home screen of your graphing calculator to model the E. coli growth.

   c. Create a scatter plot of the data and determine a mathematical model that accurately describes this set of data. Sketch your scatter plot below and write the equation for your model.

d. How many cells will we have after 1 hour? How many after 2 hours? Describe how you arrived at your solution.

e. How long will it take 1 *E. coli* cell to divide so that 1 billion cells are formed? Describe your solution strategy.