

STRAND 5: POWER, POLYNOMIAL AND RATIONAL FUNCTIONS

TOPIC 5.3: RATIONAL FUNCTIONS

Topic Notes

Mathematical focus

The mathematical focus of this topic is to understand the nature of rational functions by studying how changes in the symbolic representation of a rational function affect the graphical representation of the function. Participants will study in-depth which portion of the algebraic expression controls long-run behavior, which controls short-run behavior, why discontinuities occur, and why horizontal or oblique asymptotes occur.

Topic overview

There are eight mathematical tasks in this topic:

- 5.3.1: Rational Functions and Their Reciprocals
- 5.3.2: Functions and Their Quotients
- 5.3.3: Rational Functions and Asymptotes
- 5.3.4: Reflect and Apply: Summary
- 5.3.5: Reflect and Apply: Writing Function Rules
- 5.3.6: Reflect and Apply: Summarize and Simplify
- 5.3.7: Reflect and Apply: Create a Lesson
- 5.3.8: You're Toast, Dude!

The purpose of this topic is to expand teacher knowledge of rational functions. Using graphing calculator technology, participants will be led through a series of tasks to graph various rational functions and then to analyze the algebraic expressions to discover what in the algebraic expressions causes the particular behavior depicted by the graphs. The intent is for participants to construct meaning and to develop a mathematical framework for understanding rational functions as they progress through the various tasks. After these tasks, there is an assessment task that requires them to synthesize various rational functions, given a verbal representation. The final task in this topic requires them to create a lesson developing the same concepts in a similar fashion, but at a level suitable for high school Algebra II students.

TE_xES standards focus

TE_xES Standard II.005 Patterns and algebra. The teacher understands attributes of functions, relations, and their graphs. The beginning teacher:

(G) Uses graphs of functions to formulate conjectures of identities [e.g., $y = x^2 - 1$ and $y = (x - 1)(x + 1)$, $y = \log x^3$ and $y = 3 \log x$, $y = \sin(x+2)$ and $y = \cos x$].

TEXES Standard II.007 Patterns and algebra. The teacher understands polynomial, rational, radical, absolute value, and piecewise functions, analyzes their algebraic and graphical properties, and uses them to model and solve problems. The beginning teacher:

- (A) Recognizes and translates among various representations (e.g., written, tabular, graphical, algebraic) of polynomial, rational, radical, absolute value, and piecewise functions.
- (B) Describes restrictions on the domains and ranges of polynomial, rational, radical, absolute value, and piecewise functions.
- (C) Makes and uses connections among the significant points (e.g., zeros, local extrema, points where a function is not continuous or not differentiable) of a function, the graph of the function, and the function's symbolic representation.
- (D) Analyzes functions in terms of vertical, horizontal, and slant asymptotes.
- (E) Analyzes and applies the relationship between inverse variation and rational functions.
- (F) Solves equations and inequalities involving polynomial, rational, radical, absolute value, and piecewise functions using a variety of methods (e.g., tables, algebraic methods, graphs, use of a graphing calculator), and evaluates the reasonableness of solutions.
- (G) Models situations using polynomial, rational, radical, absolute value, and piecewise functions and solves problems using a variety of methods, including technology.

TEKS/TAKS focus

TEKS 2A.10 (A) Rational functions. use quotients of polynomials to describe the graphs of rational functions, predict the effects of parameter changes, describe limitations on the domains and ranges, and examine asymptotic behavior.

TEKS 2A.10 (B) Rational functions. analyze various representations of rational functions with respect to problem situations.

TEKS 2A.10 (C) Rational functions. determine the reasonable domain and range values of rational functions, as well as interpret and determine the reasonableness of solutions to rational equations and inequalities.

TEKS 2A.10 (D) Rational functions. determine the solutions of rational equations using graphs, tables, and algebraic methods.

TEKS 2A.10 (F) Rational functions. analyze a situation modeled by a rational function, formulate an equation or inequality composed of a linear or quadratic function, and solve the problem.

TEKS 2A.10 (G) Rational functions. use functions to model and make predictions in problem situations involving direct and inverse variation.

High School TAKS Objective 5: The student will demonstrate an understanding of quadratic and other nonlinear functions.

Materials

These tasks require only a graphing calculator.

Procedure

Review interval notation as a shorthand way to describe the domain and range of functions. Most algebra teachers will not be familiar with interval notation, as this is typically not introduced until precalculus.

Have participants work individually on Tasks 5.3.1-5.3.3. These are lengthy tasks and will probably need to be started in class and finished as homework. An alternative would be to jigsaw these tasks by dividing them among several small groups. Each group would be assigned a specific problem, or problems, to complete from these three tasks. After a specified period of time, each group would prepare a poster and then report-out to the entire group. Each group would be responsible for teaching their portion to the entire class.

Tasks 5.3.4–5.3.8 are to be done individually as assessments for this topic. Task 5.3.4 could be done at the end of class, or as homework. Tasks 5.3.5 and 5.3.6 are best completed after some reflection, and should also be assigned as homework. Task 5.3.7 should be a project extending over several days. It should be assigned at the end of this topic, with a due date approximately one week later. Task 5.3.8 is a student task, but should also be included as an assessment for the teachers.

Summary

Participants should understand that the zeros of the denominator determine where vertical asymptotes occur. If a zero occurs in both the numerator and the denominator, then an asymptote will not occur, but a removable discontinuity will occur. They should also understand that horizontal asymptotes occur when the leading-term exponents of both the numerator and denominator are the same. They should understand as well that oblique asymptotes occur when the degree of the leading-term of the numerator is one greater than the degree of the leading coefficient of the denominator.

Extensions

Extension questions are included in the Teaching Notes at appropriate places.

Assessments

Tasks 5.3.4 through 5.3.8 are intended as assessments for this topic.

Additional notes

An internet search for the topic “rational functions” will reveal a number of sites, including some with interactive graphers that allow participants to experiment with the effects of changing parameters of rational functions.