

TASK 5.3.6: REFLECT AND APPLY: SUMMARIZE AND SIMPLIFY

Solutions

Using what you have learned about rational functions, write a brief explanation of each of the following terms as you would explain them to a high school class:

(a) rational function –

Look for definitions such as the ratio of two polynomials, the quotient of two polynomials, a fraction whose numerator and denominator are both polynomials, etc. Do not accept definitions that describe rational functions as algebraic fractions. Rational functions are a specific kind of algebraic fraction.

(b) vertical asymptote –

A vertical asymptote occurs when the denominator of a rational function becomes equal to zero. As the denominator gets closer and closer to zero, we are dividing by a smaller and smaller number, which gives us a larger and larger result. As the denominator gets very close to zero, $f(x)$ gets very large, approaching either positive or negative infinity. At the point where the denominator equals zero, the graph is interrupted. Division by zero is undefined; therefore, the graph of a rational function can never cross a vertical asymptote.

(c) removable discontinuity –

A removable discontinuity occurs when the numerator and denominator both equal zero. This happens when the numerator and denominator have a common root. Since we are dividing a small number by a small number, the graph does not go off toward infinity as the denominator approaches zero. The graph simply has a hole in it at the value of x that causes the denominator to be zero.

(d) horizontal asymptote –

A horizontal asymptote occurs when the leading terms of the numerator and denominator have the same degree (same powers). As the x values grow very large, the function value tends to the fraction determined by dividing the leading coefficient of the numerator by the leading coefficient of the denominator. A horizontal asymptote is not a discontinuity in the graph; therefore, a rational function can cross its horizontal asymptote. A horizontal asymptote describes only the long-run behavior of the function.

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