

TASK 5.3.4: REFLECT AND APPLY: SUMMARY

Solutions

Write a summary of your observations about rational functions. Be sure to address how the graph is affected by the numerator and how it is affected by the denominator. Discuss when and why vertical, horizontal, and oblique asymptotes may exist in the graph of a rational function.

- *The zeros of the denominator determine where discontinuities occur in the graph of a rational function, since division by zero is undefined.*
 - *If the zero occurs only in the denominator, the discontinuity results in a vertical asymptote. A vertical asymptote is an imaginary vertical line that the function approaches when the absolute value of x approaches the zero of the denominator. The function rapidly approaches $\pm \infty$ on either side of a vertical asymptote. A rational function can never touch or cross a vertical asymptote. To do so requires that the function exist at the zero, which, by definition, it cannot.*
 - *If a zero occurs in both the numerator and the denominator, a removable discontinuity occurs instead of a vertical asymptote. A removable discontinuity is a hole in the graph of the function at the zero, because the function is undefined at this point. The function does not tend to $\pm \infty$ on either side of a removable discontinuity.*
- *The zeros of the numerator determine the x -intercepts of the graph of the rational function.*
- *The ratio of the degrees of the leading terms determines whether or not other asymptotes exist in the graph of the rational function.*
 - *If the degrees of the leading terms are equal, a horizontal asymptote will occur at $x = r$, where r is the ratio of the leading coefficients.*
 - *If the degree of the leading term of the numerator is exactly one greater than the degree of the denominator, an oblique asymptote will occur. The equation of this asymptote will be the resulting quotient when the numerator is divided by the denominator. This quotient will always be a linear equation of the form $y = mx + b$.*
 - *If the degree of the leading term of the numerator is less than the degree of the leading term of the denominator, the function will approach zero as x goes to $\pm \infty$. The x -axis ($y = 0$) will be the horizontal asymptote of the function.*

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- If the leading term of the numerator is two or more degrees higher than the degree of the denominator, there will be neither a horizontal nor an oblique asymptote. Dividing the numerator by the denominator will always result in a nonlinear function.*
- Horizontal and oblique asymptotes describe the behavior of the function in the long run (end behavior). The equation of a horizontal or oblique asymptote is the limit of the rational function, as x approaches $\pm \infty$. As such, the graph of a rational function may cross or touch a horizontal or oblique asymptote before the function finally reaches the asymptotic limit in the long run.*

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