

TASK 5.2.1: WHAT'S THE LONG RUN? WHAT'S THE SHORT RUN?**Solutions****I. Whole group discussion**

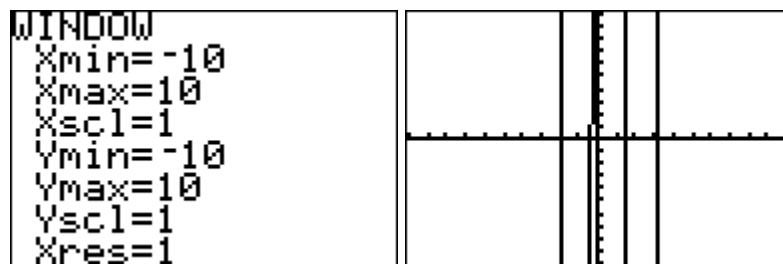
A polynomial function is a sum of power functions that have non-negative integer powers.

Graph this polynomial function and describe its long run behavior. Long run behavior is often called end behavior. It describes what the graph of the function does as x increases without bound or decreases without bound.

$$f(x) = 8x^6 - 24x^5 - 13x^4 + 42x^3 - 84x^2 + 97x + 52$$

In this task, we'll be using a graphing utility to examine the graph of polynomial functions of differing degrees. We will describe the end behavior and the short run behavior of these polynomial functions based on their degrees.

Work with the whole group to get a complete picture of the polynomial function. Start with graphing using a standard graphing window.

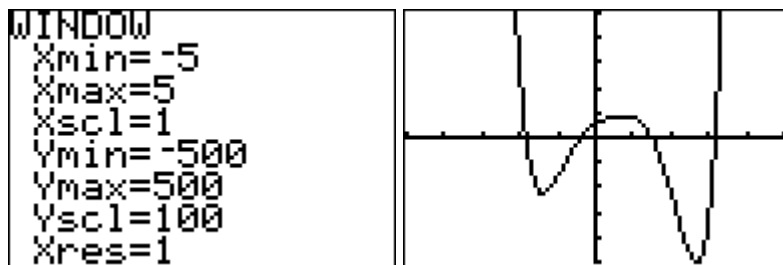


- What can we do to get a more complete picture of the polynomial function?

One option is to guess possible maximum and minimum values for y . Another option is to create a table of the function to determine the maximum and minimum possible values.

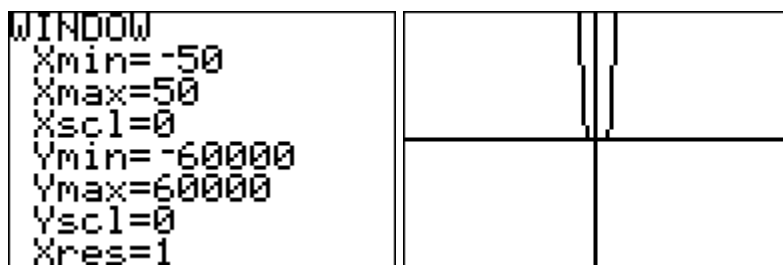
X	Y ₁	
-4	49648	
-2	258	
0	52	
2	-218	
4	6648	
6	176458	
8	1.27E6	
X=-4		

Seeing this table might suggest this new window and graph.



How do you know the graph does not turn back down?

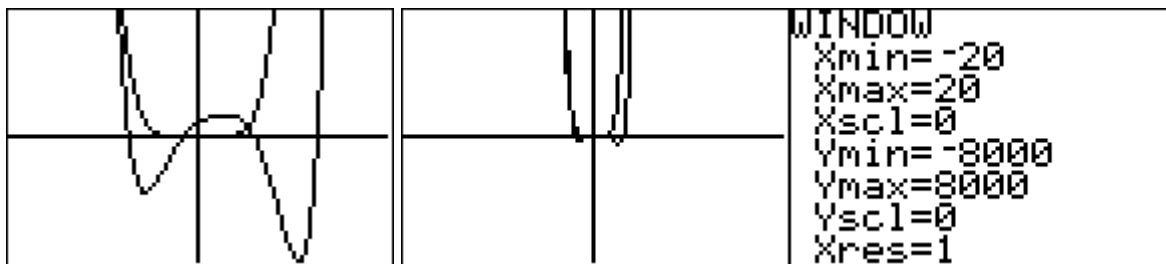
Larger and smaller values of x may be tested or the window may be increased to see the graph does not appear to turn back down.



How would you describe the long run behavior?

Long run behavior is often called **end behavior**. It describes what the graph of the function does as x grows very large positive or very large negative. Answers will vary, but participants should be guided to understand that this polynomial function's long run behavior is most affected by the power function $P(x) = 8x^6$. As x increases, this term increases much faster than any of the other terms of this polynomial function, $f(x) = 8x^6 - 24x^5 - 13x^4 + 42x^3 - 84x^2 + 97x + 52$. **This is what is often called the leading term test, i.e. the end behavior of a polynomial is completely determined by the end behavior of its leading term.**

Graphing these two functions in the original window emphasizes the differences in the short run behavior of these two functions, but increasing the window shows how alike the graphs are in the long run.



The values of these functions for very large values of x may be examined using a table or computing values on the home screen. Note that the table rounds the values, but the home screen does not.

X	Y ₁	Y ₂	10000+X	-5000+X
2000	5.1E20	5.1E20	1000	-5000
3000	5.8E21	5.8E21	Y ₁ 7.975987042E18	Y ₁ 1.250749919E23
4000	3.3E22	3.3E22	Y ₂	Y ₂
5000	1.2E23	1.2E23	8E18	1.25E23
6000	3.7E23	3.7E23		
7000	9.4E23	9.4E23		
X=1000				

- Describe how these two functions are different.

This question should lead to a discussion of the short run behavior of polynomial functions.

Short run behavior includes how many roots the polynomial has and how many turning points (relative maximum and minimum points) the function has. A **relative maximum point** is the greatest value of the function in an open interval about the x -value of the point. A **relative minimum point** is the least value of the function in an open interval about the x value of the point. To illustrate this, one may look at one of the minimum points of the graph, or by looking at a table around this x value.



The **roots or zeros of the polynomial function** are those values of x for which $f(x) = 0$. They are the x -coordinates of the x -intercepts of the polynomial function. The short run behavior of this polynomial functions is that it has four roots and three relative maximum or minimum points. Divide the participants into groups of four and assign each group one of the sets of problems. Tell participants they may need more than one viewing window. After the groups have illustrated the graphs and described the short run behavior and the long run behavior of their given degree, have each group report their conclusions to the whole group.

II. Group activities

Each group will be assigned one of these sets of polynomial functions. Graph the functions on the graphing calculator. Choose an appropriate window and sketch the graphs on large grid

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paper. Two graphs might be necessary to illustrate both the long run and short run behavior of the function. Describe the long run and short run behavior of the each function and relate it to the polynomial. The short run behavior discussion should include the number of real roots and the number of maximum and minimum points a polynomial function of the given degree may have.

Group 1:

1. $P(x) = x^6 + 4x^5 - 95x^4 - 206x^3 + 1564x^2 + 760x - 4800$
2. $P(x) = x^6 - 6x^5 - 134x^4 + 858x^3 - 1431x^2 + 864x - 1296$
3. $P(x) = x^6 - 5x^2 + 17$
4. $P(x) = -2x^6 + 6x^5 - 5x^4 + 14x^3 - 8x^2 - 24x + 12$

Group 1:

1. *6 real roots*
5 maximum or minimum points
Long run behavior like $p(x) = x^6$
2. *3 real roots (one of these is a double root)*
5 maximum or minimum points
Long run behavior like $p(x) = x^6$
3. *no real roots*
3 maximum or minimum points
Long run behavior like $p(x) = x^6$
4. *4 real roots*
3 maximum or minimum points
Long run behavior like $p(x) = -x^6$

Group 2:

1. $P(x) = x^5 - 17x^3 + 75x + 18$
2. $P(x) = 8x^5 - 444x^4 + 86x^3 - 73x^2 + 28x + 15$
3. $P(x) = x^5 - 7x^3 + 6x$
4. $P(x) = -3x^5 + 2x^4 - 8x^2 - 12$

Group 2:

1. *3 real roots*
4 maximum or minimum points
Long run behavior like $p(x) = x^5$
2. *1 real root*
2 maximum or minimum points
Long run behavior like $p(x) = x^5$

3. 5 real roots

4 maximum or minimum points

Long run behavior like $p(x) = x^5$

4. 1 real root

2 maximum or minimum points

Long run behavior like $p(x) = -3x^5$

Group 3:

1. $P(x) = x^4 - 8x^2 + 9x + 12$
2. $P(x) = -2x^4 + 15x^3 + 84x^2 - 29x - 48$
3. $P(x) = x^4 - 15x^3 - 68x^2 - 147x - 95$
4. $P(x) = x^4 - 3x^3 - 13x^2 + 51x - 36$

Group 3:

1. 2 roots

3 maximum or minimum points

Long run behavior like $p(x) = x^4$

2. 4 roots

3 maximum or minimum points

Long run behavior like $p(x) = -2x^4$

3. 2 roots

1 maximum or minimum point

Long run behavior like $p(x) = x^4$

4. 3 roots (1 double root)

3 maximum or minimum points

Long run behavior like $p(x) = x^4$

Group 4:

1. $P(x) = 5x^3 - 40x^2 + 39x + 392$
2. $P(x) = -x^3 + 27x^2 - 135x - 675$
3. $P(x) = 9x^3 + 135x^2 - 12x - 180$
4. $P(x) = -8x^3 + 9x + 1$

Group 4:

1. 1 root

2 maximum or minimum points

Long run behavior like $p(x) = 5x^3$

2. 2 roots (one double root)

2 maximum or minimum points

Long run behavior like $p(x) = -x^3$

3. 3 roots

2 maximum or minimum points
Long run behavior like $p(x) = 9x^3$

4. 3 roots

2 maximum or minimum points
Long run behavior like $p(x) = -8x^3$

Math notes

Embedded in solutions.

Scaffolding questions:

I. Whole group problem

What can we do to get a more complete picture of the polynomial function?

How do you know the graph does not turn back down?

How would you describe the long run behavior?

Describe how the two functions, $f(x) = 8x^6 - 24x^5 - 13x^4 + 42x^3 - 84x^2 + 97x + 52$ and $P(x) = 8x^6$ are different.

II. Small group problems

- What is the relationship between the degree of the polynomial function and the number of roots? the number of relative maximum and minimum points?

The number of real roots is less than or equal to the degree of the polynomial functions. If the number of real roots is less than the degree, the number of real roots is the degree minus an even number. This is because complex roots always travel in conjugate pairs. The number of relative maximum and minimum points is 1 less than the degree minus an even number.

- What does the graph do when you have a double root? How did you know you had a double root?

The graph “bounces” on the x-axis at the root, i.e. the graph is tangent to the x-axis at the root when you have a double root. You know you have a double root when a factor appears twice.

- How can you tell the difference between the graph of a polynomial function of even degree and the graph of a polynomial function of odd degree?
- Why does this difference exist?
- Why must a polynomial function of an even degree have an odd number of relative maximum and minimum points?
- Are there any situations that are not covered by your set of polynomial functions?

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- What is the effect of a negative leading coefficient?

The long run behavior is opposite of a polynomial of the same degree and a positive leading coefficient.

- Can you tell by the graph of a polynomial which degree it might be?

You can tell whether its degree is even or odd.

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