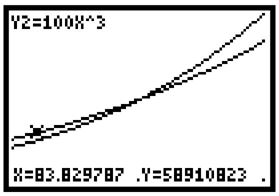
TASK 5.1.3: COMPARING FUNCTIONS: WHO WINS?

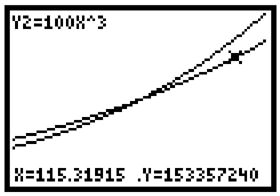
Solutions

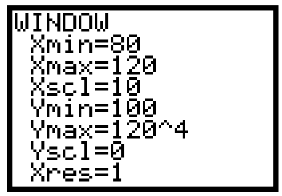
Let's race. Who "wins" in the long run? Support your answer numerically and graphically.

1. $f(x) = 100x^3$ versus $g(x) = x^4$.

The two functions are equal when $x = 100$. *This is the point after which* $y = x^4$ *overtakes* $y = 100x^3$.





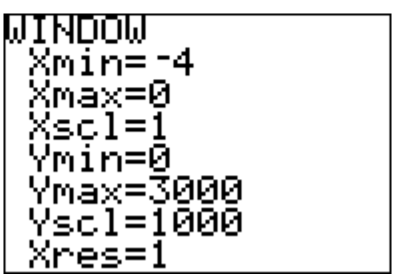


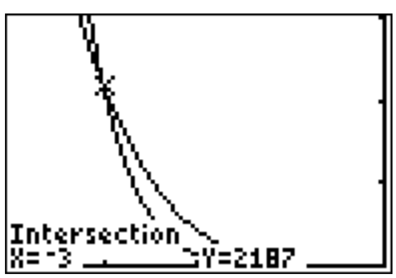
Scaffolding questions:

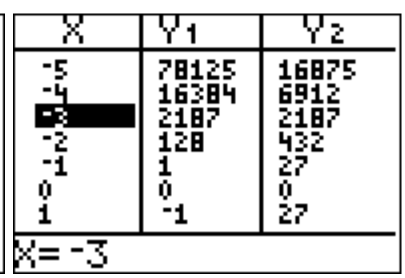
- What is the behavior of these functions for very large negative values of x (i.e. as x decreases without bound)?
  For very large negative values, the even exponent makes $y = x^4$ *always positive and the odd exponent means* $y = 100x^3$ *is negative.*
- What has more effect, the coefficient, k, or the exponent p in the long run?
- When you found where f and g intersect, what equation(s) were you solving?
  Connect $100x^3 = x^4$ *and* $x^4 - 100x^3 = 0$, $x^3(x - 100) = 0$.
- Which function "wins" as $x \to -\infty$? That is, does x^4 get big positive faster than $100x^3$ gets big negative, or vise versa?
  (Try graphing $y = x^4 - 100x^3$ *for large negative values of x.)*

2. $f(x) = -x^7$ versus $g(x) = 27x^4$

$y = -x^7$ *dominates* $y = 27x^4$ *for large negative values. For negative values greater than -3,* $y = 27x^4$ *dominates* $y = -x^7$ *(Find where the two graphs intersect by solving the equation* $27x^4 = -x^7$*.) To determine when the change occurs the table could be examined or the intersect feature on the calculator may be used.*







X	Y1	Y2
-5	78125	16875
-4	16384	6912
-3	2187	2187
-2	128	432
-1	1	27
0	0	0
1	-1	27

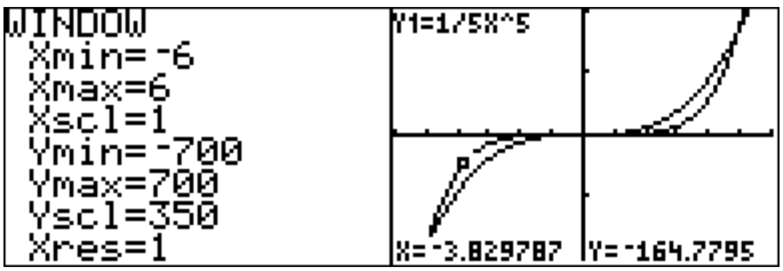
X= -3

Scaffolding questions:

- How did you decide what graphing window to use?
- As $x \to +\infty$, which function is changing fastest: f as it decreases or g as it increases?
  To answer this question, examine the difference (f – g) of the two functions. The gap between the functions gets bigger and bigger negative as x increases, so f "wins".

3. $f(x) = \frac{1}{5}x^5$ versus $g(x) = 5x^3$**.**

In the long run, $f(x) = \frac{1}{5}x^5$ *dominates* g(x) = $5x^3$*. The two functions are equal at* x = *-5 and at* x = *5. To see this, solve the equation* f(x) = g(x)*. Use two different windows to show the relative behavior of the functions.*



Teaching notes
This task will build the ideas to be used in the leading term test for determining the long run behavior of a polynomial. Take a moment to discuss what it means for one function to "win" when compared to another. Assign each group one of the three exercises. Ask them to justify their reasoning using large grid paper and to present their findings to the group.

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