

TASK 5.1.1: INTRODUCTION TO POWER FUNCTIONS**Solutions**

One of Johannes Kepler's three laws of planetary motion states that the square of the period, P , of a body orbiting the sun is directly proportional to the cube of its average distance, d , from the sun. Symbolically, $P^2 = kd^3$.

1. Solve for P .

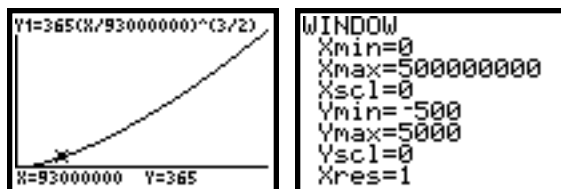
$$P = \sqrt{kd^3}$$

2. For earth, use a period of 365 days and a distance from the sun of approximately 93,000,000 miles. Write the function for the period P in terms of the average distance, d , from the sun.

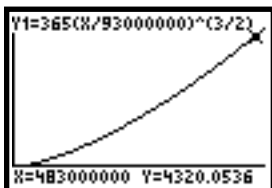
Solve for k to find $k = \frac{365^2}{93,000,000^3}$. Substitute k back in $P = \sqrt{kd^3}$ to get

$$P = 365 \left(\frac{d}{93,000,000} \right)^{\frac{3}{2}}$$

3. Find a good viewing window for the situation. Sketch the graph.



4. Given that the planet Jupiter has an average distance from the sun of 483,000,000 miles, how long in Earth days is a Jupiter year?



A Jupiter year is approximately 4,320 earth days.

Math notes

Participants may be uncomfortable with the idea of raising a variable to a fractional, or even irrational, power. The important thing for the topics that follow, Polynomials and Rational Functions, is that the participants get a good feeling for the general shapes of the basic power functions which have natural number powers.

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Teaching notes

Write a general power function on the transparency:

$$y = kx^p, \text{ where } k \text{ and } p \text{ are any real numbers.}$$

Ask if participants can think of examples of real situations that are modeled by power functions?

Possible answers:

Area, A , in terms of the radius, r : $A = \pi r^2$

The time it takes to travel 300 miles depends on the rate, r :

$$t = \frac{300}{r} = 300r^{-1}.$$

The side of a square, s , depends upon the area of the square, A :

$$s = \sqrt{A} = A^{\frac{1}{2}}.$$

Discuss the scenario in Task 5.1.1. k is referred to as the proportionality constant. Have participants work on the task in groups. Circulate and ask guiding questions.

Scaffolding questions

What is the advantage of solving the relationship for one variable in terms of the other?

Does it make sense to graph the function for values of d or P which are negative?

Extension question

Do you have enough information to find out how long an Earth year is in Jupiter days?

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(Teacher)

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