TASK 4.2.3: TRUTH OR CONSEQUENCES

Solutions

Graph each side of the following equations separately and decide if each statement is Always true, Sometimes true, or Never true. If the equation is sometimes true, state when this occurs.

Use the table feature of you calculator to support your conjecture and give algebraic support as well.

1. $\sqrt{x^2+9} = x+3$ — S (x = 0)
2. $\sqrt{x-2} = \sqrt{x}-2$ — N
3. $\sqrt{x^2} = x$ — S ($x \geq 0$)
4. $\left(\sqrt{x}\right)^2 = x$ — S ($x \geq 0$)
5. $\sqrt{16x^2} = 4\sqrt{x}$ — A
6. $\sqrt{-x} = -\sqrt{x}$ — S (x = 0)
7. $\sqrt{x^4} = x^2$ — A
8. $\sqrt{\frac{x}{9}} = \frac{\sqrt{x}}{3}$ — A
9. $3\sqrt{x-2} = \sqrt{3x-6}$ — S (x = 2)
10. $\sqrt{x}+1 = \sqrt{x+1}$ — S (x = 0)

Math notes

This task highlights misconceptions students have about taking square roots. By looking at algebraic expressions as functions and investigating their graphs, they can see exactly what it means that an expression is always, never, or sometimes true.

Teaching notes

For the equations given on the activity sheet, students graph the left and right sides as separate functions. Then they decide whether each statement is always true, sometimes true of never true. Students should be told to use the table feature of the calculator to reinforce their conjectures.

Some discussion may be needed on what is means to prove something is always true. Students should give algebraic support for the conjectures.

Assessment

Ask the participants to write a reflection in which they discuss how to explain to their students that $\sqrt{x^2+16} \neq x+4$ (except when x=0) and that $\sqrt{x-3} \neq \sqrt{x}-3$ in three or more ways. They should be asked to keep in mind the various types of learners and multiple representations.

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