

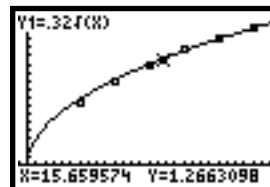
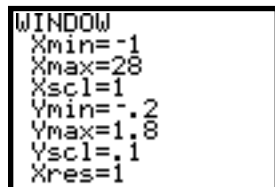
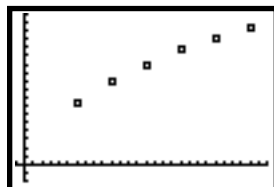
TASK 4.2.2: THE PENDULUM SWINGS**Solutions**

Let's investigate the relationship between the period and the length of a simple pendulum.

1. Construct a simple pendulum by tying a string 32 inches long to a washer. Hold it steady or attach it to something that will allow it to swing freely.
2. Swing the pendulum so that the weight swings in a short arc of about 10 degrees. Time 5 complete swings of the pendulum (a complete swing is back *and* forth). Divide by 5 to find the period.
3. Repeat the experiment for the different lengths listed in the table.

Sample Data:

Length of the Pendulum	Period
26	1.632
22	1.524
18	1.370
14	1.192
10	1.004
6	0.7367



4. Set up a scatter plot of the data. Use the function grapher on your calculator and guess and check to find a model.

What length of pendulum do you need to get a period of 0.5 seconds?

5. Using your model, write the equation whose solution answers the question above.

$$0.32\sqrt{x} = 0.5$$

6. Solve the equation algebraically, step by step. At each step, solve the new equation graphically. Sketch each resulting graph. Discuss what you find.

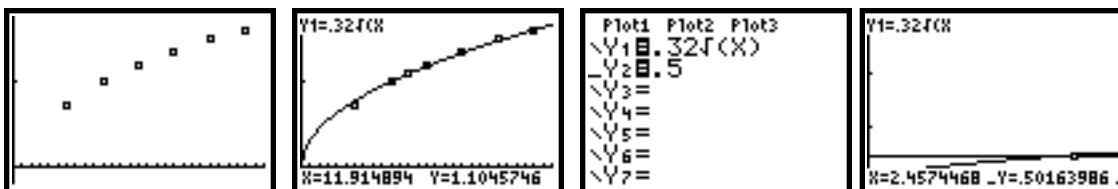
$$0.32\sqrt{x} = 0.5$$

$$\sqrt{x} = \frac{0.5}{0.32}$$

$$(\sqrt{x})^2 = \left(\frac{0.5}{0.32}\right)^2$$

Algebra II: Strand 4. Square Root Functions; Topic 2. Applications; Task 4.2.2

$$x = \left(\frac{0.5}{0.32} \right)^2 \approx 2.44$$

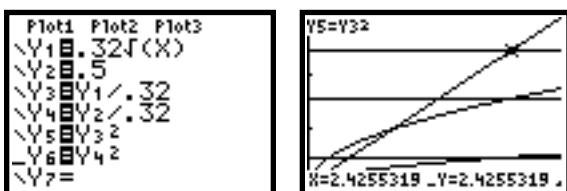


- What is the first step in the algebraic solution?
Divide both sides by 0.32.
- What will the graph look like?
Dividing both sides by 0.32 is a vertical stretch because dividing by 0.32 is the same as multiplying by 3.125.
- What stays the same?
The x-values of the intersections are the same.



- What is the second step in the algebraic solution?
Square both sides.
- How will that affect the graphs?
 $(\sqrt{x})^2$ is the line $y = x$, $x \geq 0$. $y = \left(\frac{0.5}{0.32} \right)^2$ is a horizontal line that is higher than $y = \left(\frac{0.5}{0.32} \right)$.

- What stays the same?
The x-values of the intersections are the same. All of the solutions lie on the vertical line $x =$ solution.



Algebra II: Strand 4. Square Root Functions; Topic 2. Applications; Task 4.2.2

The law for the relationship between the period and the length of a simple pendulum is

$P = 2\pi\sqrt{\frac{l}{a}}$, where a is the acceleration due to gravity, $32\frac{\text{feet}}{\text{sec}^2}$. Rewrite the law as

$P = \frac{2\pi}{\sqrt{a}}\sqrt{l} = \frac{2\pi}{\sqrt{32(12)}}\sqrt{l}$. Note that our guess and check equation is $y = 0.32\sqrt{x}$ and

A calculator screen showing the calculation $2\pi / (\sqrt{12 \cdot 32})$ resulting in the decimal value $.3206374575$.

7. Use your pendulum to check your answer.

Math notes

The law for the relationship between the period and the length of a simple pendulum is

$P = 2\pi\sqrt{\frac{l}{a}}$, where a is the acceleration due to gravity, $32\frac{\text{feet}}{\text{sec}^2}$. Since the units of length in this

task are inches, we convert $32\frac{\text{feet}}{\text{sec}^2}$ to $32\frac{\text{feet}}{\text{sec}^2} = 32\left(\frac{12\text{ inches}}{1\text{ foot}}\right)\frac{\text{feet}}{\text{sec}^2} = 32 \cdot 12\frac{\text{inches}}{\text{sec}^2}$. Thus we

can rewrite the law as $P = \frac{2\pi}{\sqrt{a}}\sqrt{l} = \frac{2\pi}{\sqrt{32(12)}}\sqrt{l} = \frac{\pi}{4\sqrt{6}}\sqrt{l}$.

Teaching notes

Have students construct their simple pendulums with string and washers and then collect the data. Give one group the overhead calculator and have them enter their data into it for you to use with the whole group in the next part of the activity. Circulate and ensure that students swing their pendulums in a small arc, about 10° . This is important because the rule that we are aiming for only works for pendulums swinging in small arcs.

It would be wise to come to class with some relatively good data in case one of the groups can't get numbers that make any sense.

Scaffolding questions:

- How do you know a quadratic function is not a reasonable model?

TASK 4.2.2: THE PENDULUM SWINGS — STUDENT VERSION

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3. Repeat the experiment for the different lengths listed in the table.

Length of the Pendulum	Period
26 inches	
22 inches	
18 inches	
14 inches	
10 inches	
6 inches	

4. Set up a scatter plot of the data. Use the function grapher on your calculator and guess and check to find a model.

What length of pendulum do you need to get a period of 0.5 seconds?

5. Using your model, write the equation whose solution answers the question above.
6. Solve the equation algebraically, step by step. At each step, solve the new equation graphically. Sketch each resulting graph. Discuss what you find.
7. Use your pendulum to check your answer.