

TASK 4.2.1: I CAN SEE FOREVER ...**Solutions**

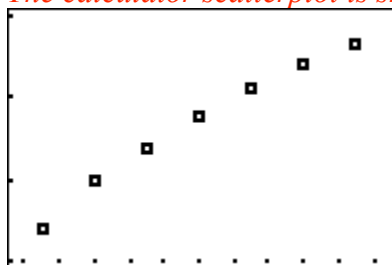
Because you really enjoyed the balloon ride you took two years ago at the county fair, you have decided to take another one this year. This time, you take your binoculars with you to look around while you are airborne. As the balloon rises you use a device called a GPS(Global Positioning System) to determine your height in meters, h , and the distance in kilometers, d , to the farthest object that you can see. The table below compares these two measurements.

Height in meters, h	15	30	45	60	75	90	105
Distance in kilometers, d	13.833	19.562	23.959	27.665	30.931	33.883	36.598

- Given that the distance to an object depends on the height of the balloon, make a scatterplot of the information in the table.

We would first have to be at a certain height and then find the distance to an object. The distance you see is dependent on how high you are.

The calculator scatterplot is shown below.



Scaffolding Questions for Question 1:

- Do you think the balloon ride is in an area with lots of hills and mountains? Why?*
- Why can't you "see forever"?*
- Does there appear to be a strong pattern to the points in the plot?*

- What parent function has the same approximate shape? Discuss the domain of the situation and how this will help you determine which function is most reasonable.

It might appear that a linear, quadratic, logarithmic, or square root function could be used as a model. The model is not linear because there does not appear to be a constant rate of change. The shape of the data does not match a quadratic function, because the data appears to be concave down and not concave up. That would mean the data would have to eventually decrease as h increases, which doesn't make sense. A logarithmic function would not be a good model because it would have negative values between zero and one and the graph begins to increase slowly. So, this function doesn't seem like a good model. A square root function seems most reasonable because the shape of the data matches the shape of the graph of a square root function.

Scaffolding Questions for Questions 2 & 3:

- *Looking at the numbers in the table, does the relationship appear to be linear? How would you check without looking at the graph?*
- *Sketch a parabola through the points to illustrate why, over a reasonably large domain, it would not be a good model.*
- *Models are built around assumptions. What assumptions are you making in this problem?*

3. Determine a function that models this data.

The general form of the square root function is $y = a\sqrt{x-h} + k$. The scatterplot indicates that there may not be a horizontal or vertical shift, so an attempt to find an equation of the form $y = a\sqrt{x}$ is made. Since the dependent variable is distance, d , and the independent variable is height, h , the equation will be of the form $d = a\sqrt{h}$. The point $(15, 13.833)$ is used to determine a possible value for a .

$$d = a\sqrt{h}$$

$$13.833 = a\sqrt{15}$$

$$a = \frac{13.833}{\sqrt{15}}$$

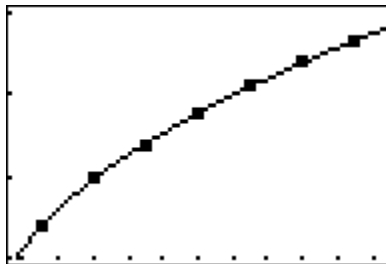
$$a \approx 3.572$$

Checking another value for h , 75, gives $3.572\sqrt{75} = 30.934$, which is reasonably close to the value in the table, 30.931.

Using the list feature of the calculator to check shows that the model appears reasonably correct.

L1	L2	3	L1	L2	L3	3
15	13.833	-----	15	13.833	13.833	
30	19.562		30	19.562	19.319	
45	23.959		45	23.959	23.661	
60	27.665		60	27.665	27.322	
75	30.931		75	30.931	30.546	
90	33.883		90	33.883	33.462	
105	36.598		105	36.598	36.143	
L3 = 3.5272√(L1)			L3(1) = 13.66078685...			

Checking by looking at a graph of $d = 3.572\sqrt{h}$ with the scatter plot shows a reasonable fit.



4. If the balloon is 800 meters high, what is the farthest distance that you would be able to see?

$$d = 3.572\sqrt{800}$$

$$d = 101.031\text{km}$$

Scaffolding Questions for Question 4:

- *When considering how high you have to be to see a building 800 meters away, does the height of the building matter?*

5. A building is 16 kilometers away. To what height does the balloon have to rise for the building to be visible?

$$16 \leq 3.572\sqrt{h}$$

$$h \geq 20.064\text{m}$$

You would have to be at least 20.064 m in the air.

Math notes

Participants may be tempted immediately use the regression features on their calculators to find the “best” fit function. Instead, to increase their understanding of parent functions, ask participants to use some trial and error and their knowledge of parent functions to find an appropriate function for the situation.

Teaching notes

Ask the participants to work in groups on this task. As you circulate the classroom, consider posing the following questions as you listen to participant conversations.

Have participants consider the following extension questions for homework. Start the following session with a discussion on their answers to the questions.

Additional Extension Questions:

- How large would the domain be if you were in a rocket? Would the range continue to follow this model?
- At what point do you think this relationship will no longer hold?
- Is this problem actually about inequalities?

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1. Given that the distance to an object depends on the height of the balloon, make a scatterplot of the information in the table.
2. What parent function has the same approximate shape? Discuss the domain of the situation and how this will help you determine which function is most reasonable.
3. Determine a function that models this data.
4. If the balloon is 800 meters high, what is the farthest distance that you would be able to see?
5. A building is 16 kilometers away. To what height does the balloon have to rise for the building to be visible?