

STRAND 4: SQUARE ROOT FUNCTIONS

TOPIC 4.1: INVERSE FUNCTIONS

Topic Notes

Mathematical focus

Participants will see the interplay between square root functions and quadratic functions and how their inverse relationship fits together with regard to both their graphs and formulas.

Topic overview

This topic includes 2 tasks:

Task 4.1.1: Square Root Functions as Inverse Functions

Task 4.1.2: Undoing a Square Root and Undoing a Square?

In Task 1.4.1, participants investigated inverse functions and were introduced to the fact that the composition of inverse functions is equivalent to the identity function ($f(x) = x$). In Task 4.1.1, the participants will use a transformational approach to understand the relationship between the formula for a quadratic function and its inverse, a square root function. This task will tie together the necessity of restricting a function's domain with the resulting corresponding inverse function. Task 4.1.2 reinforces the notion that squaring a function followed by taking the square root (or vice versa) does not necessarily result in the identity function (that is, the result is the original function) unless attention is paid to the appropriate correspondence of the domain and range of the "squaring function" and the "radical function."

TE_xES standards focus

TE_xES Standard II.004 Patterns and algebra. The teacher uses patterns to model and solve problems and formulate conjectures. The beginning teacher:

(A) Recognizes and extends patterns and relationships in data presented in tables, sequences, or graphs.

TE_xES Standard II.005 Patterns and algebra. The teacher understands attributes of functions, relations, and their graphs. The beginning teacher:

(A) Understands when a relation is a function.

(B) Identifies the mathematical domain and range of functions and relations and determines reasonable domains for given situations.

(E) Applies basic transformations [e.g., $k f(x)$, $f(x) + k$, $f(x - k)$, $f(kx)$, $|f(x)|$] to a parent function, f , and describes the effects on the graph of $y = f(x)$.

(F) Performs operations (e.g., sum, difference, composition) on functions, finds inverse relations, and describes results symbolically and graphically.

TE_xES Standard II.007 Patterns and algebra. The teacher understands polynomial, rational, radical, absolute value, and piecewise functions, analyzes their algebraic

and graphical properties, and uses them to model and solve problems. The beginning teacher:

(A) Recognizes and translates among various representations (e.g., written, tabular, graphical, algebraic) of polynomial, rational, radical, absolute value, and piecewise functions.

(B) Describes restrictions on the domains and ranges of polynomial, rational, radical, absolute value, and piecewise functions.

(F) Solves equations and inequalities involving polynomial, rational, radical, absolute value, and piecewise functions using a variety of methods (e.g., tables, algebraic methods, graphs, use of a graphing calculator), and evaluates the reasonableness of solutions.

TEExES Standard V.018 Mathematical processes and perspectives. The teacher understands mathematical reasoning and problem solving. The beginning teacher:

(E) Understands the problem-solving process (i.e., recognizing that a mathematical problem can be solved in a variety of ways, selecting an appropriate strategy, evaluating the reasonableness of a solution).

TEExES Standard V.019 Mathematical processes and perspectives. The teacher understands mathematical connections both within and outside of mathematics and how to communicate mathematical ideas and concepts.

(A) Recognizes and uses multiple representations of a mathematical concept (e.g., a point and its coordinates, the area of a circle as a quadratic function of the radius, probability as the ratio of two areas, area of a plane region as a definite integral).

TEKS/TAKS focus

TEKS 2A.6 Quadratic and square root functions. The student understands that quadratic functions can be represented in different ways and translates among their various representations. The student is expected to:

(A) determine the reasonable domain and range values of quadratic functions, as well as interpret and determine the reasonableness of solutions to quadratic equations and inequalities; and

(B) relate representations of quadratic functions, such as algebraic, tabular, graphical, and verbal descriptions.

TEKS 2A.7 Quadratic and square root functions. The student interprets and describes the effects of changes in the parameters of quadratic functions in applied and mathematical situations. The student is expected to:

(B) use the parent function to investigate, describe, and predict the effects of changes in a , h , and k on the graphs of $y = a(x - h)^2 + k$ form of a function in applied and purely mathematical situations.

TEKS 2A.9 Quadratic and square root functions. The student formulates equations and inequalities based on square root functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. The student is expected to:

- (A) use the parent function to investigate, describe, and predict the effects of parameter changes on the graphs of square root functions and describe limitations on the domains and ranges;
- (B) relate representations of square root functions, such as algebraic, tabular, graphical, and verbal descriptions;
- (C) determine the reasonable domain and range values of square root functions, as well as interpret and determine the reasonableness of solutions to square root equations and inequalities;
- (G) connect inverses of square root functions with quadratic functions.

High School TAKS Objective 1: The student will describe functional relationships in a variety of ways.

High School TAKS Objective 2: The student will demonstrate an understanding of the properties and attributes of functions.

High School TAKS Objective 5: The student will demonstrate an understanding of quadratic and other nonlinear functions.

Materials

	4.1.1: Square Root Functions as Inverse Functions	4.1.2: Undoing a Square Root and Undoing a Square?
Graphing calculator	x	x
Patty paper	x	

Procedure

Ask participants to work in groups, and have them discuss their answers as a large group as they work through the tasks.

Summary

The big idea here is that quadratic and square root functions are inverses of each other; however, the particular functions are dependent upon the domain restrictions one chooses.

Extensions

In what type of a situation would it make most sense to use the “bottom half” square root function, h , as the inverse of g ?

Assessment

It would be most appropriate to have participants complete the teacher journal for this task.

	Teacher use only	Modify for students	Ready for students
Task 4.1.1		*	
Task 4.1.2	*		