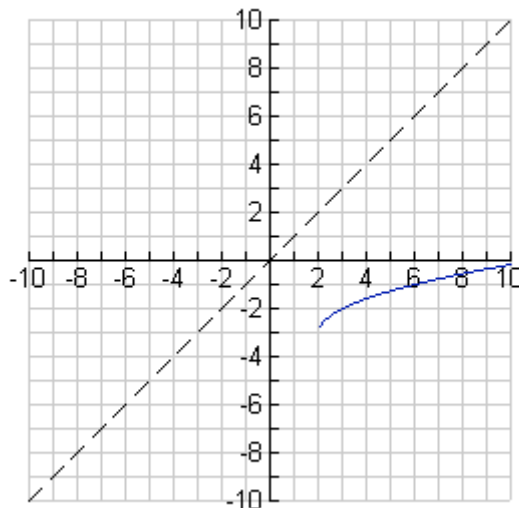


TASK 4.1.1: SQUARE ROOT FUNCTIONS AS INVERSE FUNCTIONS**Solutions**

Let $f(x) = \sqrt{x-2} - 3$.

- Graph f below by transforming the parent function. Use your calculator to check your graph.



The parent function is $y = \sqrt{x}$. To get the graph of f , translate to the right 2 and down 3.

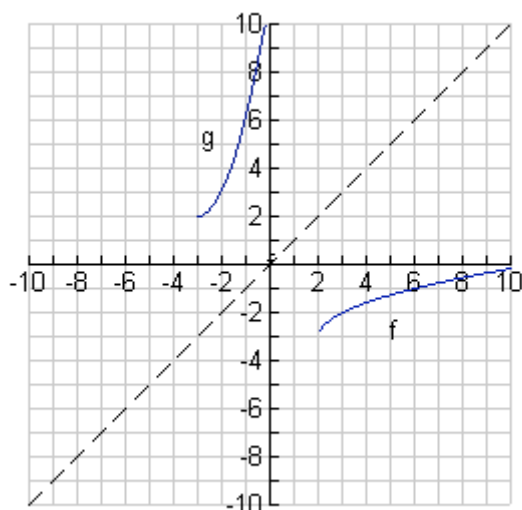
- What are the domain and range of f ?

From the graph, we can see that the domain of f is $x \geq 2$ and the range is $y \geq -3$. This makes sense since we know we can't take the square root of a negative number, so $x - 2 \geq 0$, and since we know the range of the square root parent function is all non-negative numbers, the range gets shifted down to start at -3 .

- Describe f . What is interesting about the function?

f is increasing, one-to-one, and concave down.

- Using patty paper, reflect the graph of f across the line $y = x$ (the dashed line on the graph.) Call this new function g . (Are you sure it is a function?)



5. What are the domain and range of g ?

The domain of g is $x \geq -3$ and the range is $y \geq 2$.

6. What type of function does g look like, i.e. what would the parent function be?

g looks like part of a parabola, with the vertex at $(-3, 2)$

7. Using transformations of the parent function, find a formula for g .

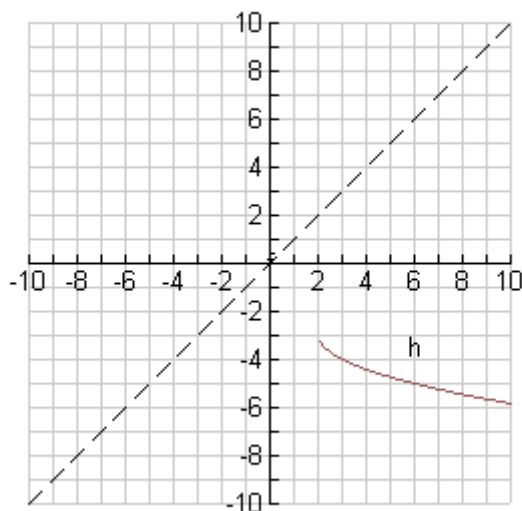
Starting with $y = x^2$, translate left 3 and up 2. Thus the formula is $g(x) = (x + 3)^2 + 2$.

8. Can you make any connections between the formulas for f and g and their inverse relationship?

Notice that whereas f had a horizontal shift right 2 (subtracting from the x), g has a vertical shift up 2 (adding to the y). While f had a vertical shift down 3 (subtracting from the y), g has a horizontal shift left 3 (adding to the x). This makes sense because f is the inverse of g (and vice versa). It is important to note that g is only defined for $x \geq -3$, even though the parabola $y = (x + 3)^2 + 2$ is defined for all x values.

Now let $h(x) = -\sqrt{x-2} - 3$.

9. Graph h below using transformations of the parent function.



Start with the parent function $y = \sqrt{x}$. Reflect about the x-axis, shift right 2, and shift down 3.

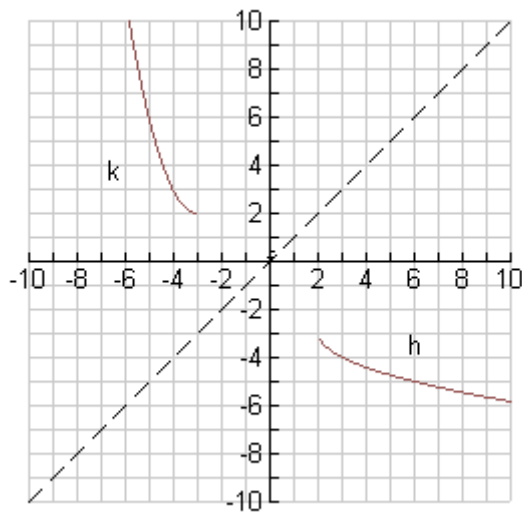
10. What are the domain and range of h ?

From the graph, we can see that the domain of f is $x \geq 2$ and the range is $y \leq -3$. This makes sense since we know we can't take the square root of a negative number, so $x - 2 \geq 0$, and since we know the range of the square root parent function is all non-negative numbers, reflecting the function about the x-axis and shifting it down 3 makes the range hit all numbers less than or equal to -3.

11. Describe h . What are the characteristics of this function?

h is decreasing, one-to-one, and concave up.

12. Using patty paper, reflect h across the line $y = x$. Call this new function k .



13. What are the domain and range of k ?

The domain of k is $x \leq -3$ and the range is $y \geq 2$.

14. Using transformations of the appropriate parent function, find a formula for k .

k looks like half of a parabola with vertex $(-3, 2)$. Starting with $y = x^2$, translate left 3 and up 2. Thus the formula is $k(x) = (x + 3)^2 + 2$.

15. How do the formulas for h and k relate to each other?

Again we see how the inverse relationship affects the formulas. Notice that whereas h had a horizontal shift right 2 (subtracting from the x), k has a vertical shift up 2 (adding to the y). While h had a vertical shift down 3 (subtracting from the y), k has a horizontal shift left 3 (adding to the x). The reflection across the x -axis that is necessary when graphing h shows up in the domain of k , but not in the formula, per se.

16. Summarize your findings. In particular, what did you notice about the formulas for g and k ? Why is this the case?

g and k have the same formulas, though their domains are disjoint. This arises because the function $y = (x + 3)^2 + 2$ is not one-to-one, so to define an inverse function for it, one needs to restrict the domain. f and h are the inverses corresponding to the two logical domain restrictions.

Math notes

Be sure to point out to the participants that even though two functions may have the same formula, if their domains are different, then the functions are different.

December 14, 2004. *Ensuring Teacher Quality: Algebra II*, produced by the Charles A. Dana Center at The University of Texas at Austin for the Texas Higher Education Coordinating Board.

Teaching notes

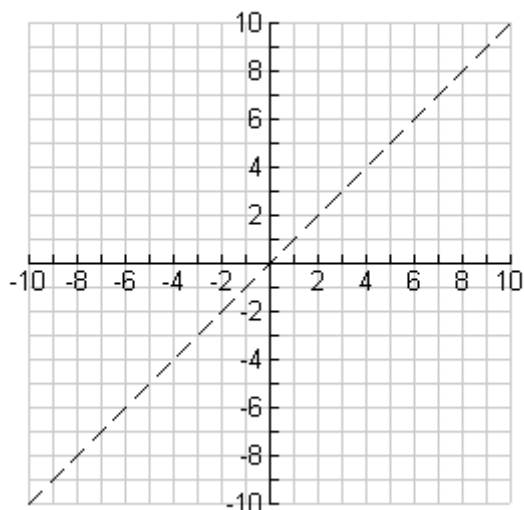
Try to emphasize the transformational ideas. Participants may immediately jump to finding the formula for the inverse function by using an algorithm, but then they might miss the point that a transformation in the original function's x direction gets undone in the inverse function's y direction, and that a transformation in the original function's y direction gets undone in the inverse function's x direction.

Note: To demonstrate the reflection process about the line $y = x$ on the overhead, it works well to use a transparency and crease it along the line.

TASK 4.1.1: SQUARE ROOT FUNCTIONS AS INVERSE FUNCTIONS

Let $f(x) = \sqrt{x-2} - 3$.

1. Graph f below by transforming the parent function. Use your calculator to check your graph.



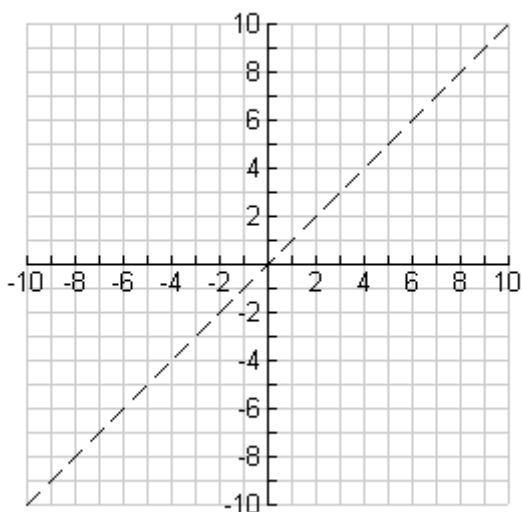
2. What are the domain and range of f ?
3. Describe f . What is interesting about the function?
4. Using patty paper, reflect the graph of f across the line $y = x$ (the dashed line on the graph.) Call this new function g . (Are you sure it is a function?)
5. What are the domain and range of g ?
6. What type of function does g look like, i.e. what would the parent function be?

7. Using transformations of the parent function, find a formula for g .

8. Can you make any connections between the formulas for f and g and their inverse relationship?

Now let $h(x) = -\sqrt{x-2} - 3$.

9. Graph h below using transformations of the parent function.



10. What are the domain and range of h ?

11. Describe h . What are the characteristics of this function?

