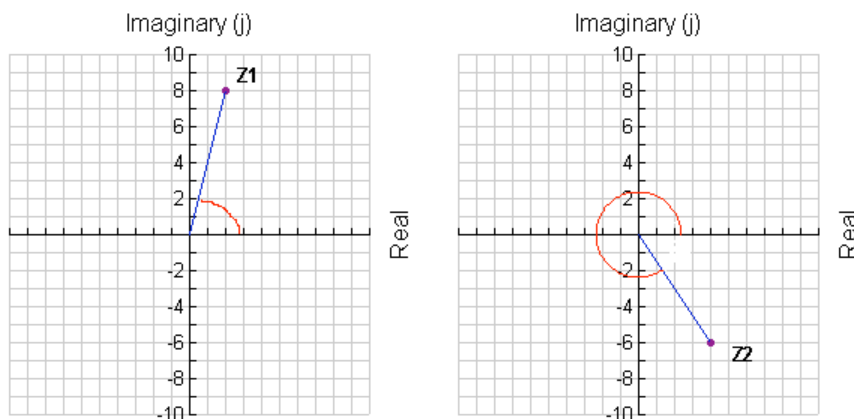


**TASK 3.3.3: "SHOCKING" NUMBERS****Solutions**

For the problem setup, please refer to the student worksheet that follows this section.

1. The impedance in *part 1* of an AC circuit is  $Z_1 = 2 + j8$  ohms, and the impedance in *part 2* of the circuit is  $Z_2 = 4 - j6$  ohms.
  - a. Using the complex plane, plot  $Z_1$  and  $Z_2$ . On your graph, indicate the phase shift for each impedance.



- b. Find the magnitude of the impedance for each of  $Z_1$  and  $Z_2$ .

$$|Z_1| = \sqrt{68}, |Z_2| = \sqrt{52}$$

- c. Suppose the two parts of the circuit are connected in series. Find the equivalent impedance ( $Z_{\text{series}}$ ) in the entire circuit, where  $Z_{\text{series}} = Z_1 + Z_2$ .

$$Z_{\text{series}} = 6 + j2$$

- d. Suppose the two parts of the circuit are connected in parallel. Find the equivalent impedance ( $Z_{\text{parallel}}$ ) in the entire circuit, where  $Z_{\text{parallel}} = \frac{Z_1 \cdot Z_2}{Z_1 + Z_2}$ .

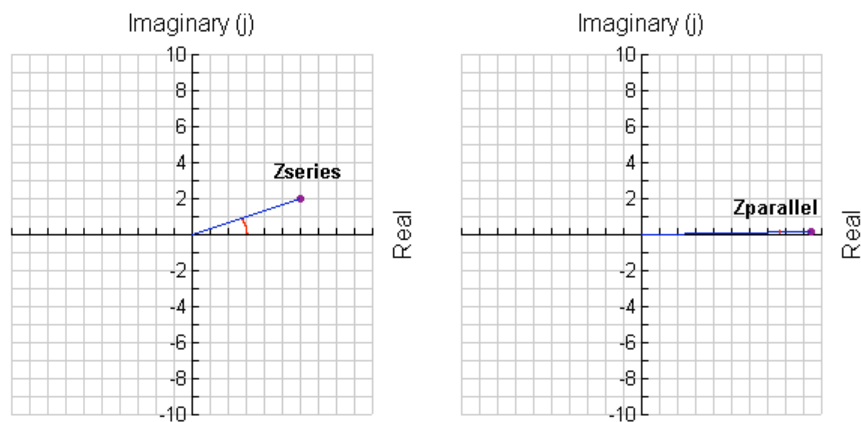
$$Z_{\text{parallel}} = \frac{47}{5} + j\frac{1}{5}$$

Algebra II: Strand 3. Quadratic Functions; Topic 3. Applications; Task 3.3.3

- e. Find the magnitude of the impedance for each of  $Z_{\text{series}}$  and  $Z_{\text{parallel}}$ .

$$|Z_{\text{series}}| = 2\sqrt{10}, \quad |Z_{\text{parallel}}| = \frac{\sqrt{2210}}{5}$$

- f. Plot  $Z_{\text{series}}$  and  $Z_{\text{parallel}}$  and indicate the phase shift.



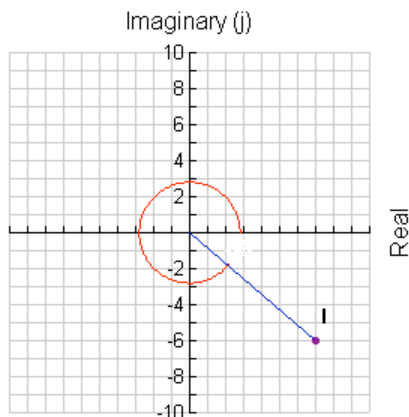
2. The voltage in an AC circuit is  $45 + j10$  volts and the impedance is  $3 + j4$  ohms.
- a. What is the current?

$$I = 7 - j6$$

- b. What is the amplitude of the current?

$$|I| = \sqrt{85}$$

- c. Plot the current on the complex plane and indicate the phase shift.



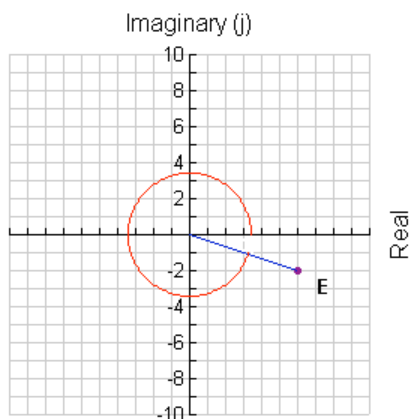
3. The current in an AC circuit is  $4 + j2$  amps and the impedance is  $1 - j$  ohms.
- a. What is the voltage?

$$E = 6 - j2$$

- b. What is the amplitude of the voltage?

$$|E| = 2\sqrt{10}$$

- c. Plot the voltage on the complex plane and indicate the phase shift.



### Math notes

The main idea here is that for an AC circuit, current, voltage, and impedance are multidimensional rather than scalar quantities. If participants are familiar with vectors, they may recognize that complex numbers can be visualized in a way similar to two-dimensional vectors. This gives a good example to show why one would want to simplify one complex number divided by another.

If the participants are familiar with polar coordinates, this is an opportunity to show the polar form of a complex number and how arithmetic in that form can be more convenient in certain situations (multiplication and division).

**Teaching notes**

This is a good opportunity to get the participants thinking about what other quantities in their daily lives are multidimensional.

Work through the example as a group and make sure all the vocabulary is understood, as this can be a stumbling block to students who don't know much about electricity.

**Technology notes**

Students can do complex arithmetic on their calculators. You might suggest they verify their answers using the calculator.  $i$  can be found by pressing [ $2^{\text{nd}}$ ] [decimal point]. It is not necessary to switch the mode of your calculator to complex mode. Emphasize that enclosing complex numbers within parentheses is essential when doing complex arithmetic on the calculator.

### TASK 3.3.3: “SHOCKING” NUMBERS

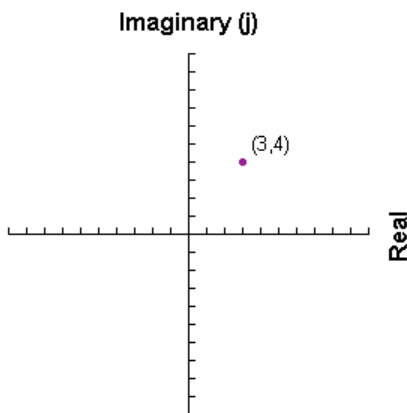
We see complex numbers appear when we use the quadratic formula to find the roots of a quadratic function. But does anyone other than mathematicians use complex numbers in the real world?

Here’s one example. When electrical engineers analyze alternating current (AC) circuits, like those found in the electrical system in your house, they find that the quantities of voltage, current, and resistance (called **impedance** in AC) are not the familiar one-dimensional numerical quantities that are used when measuring direct current (DC) circuits (like in a battery). Even in an AC circuit where the frequency of the circuit is constant, they can’t simply measure the “size” of these quantities, they also need to take into account the **phase shift**. The phase shift compares how the quantity is alternating relative to a fixed reference wave.

In order to analyze AC circuits, for the voltage, current, and impedance it is necessary to represent both the “size” (**amplitude** or **magnitude**) and the phase shift all at once. In order to accomplish this task, complex numbers are used to express the two dimensions at one time.

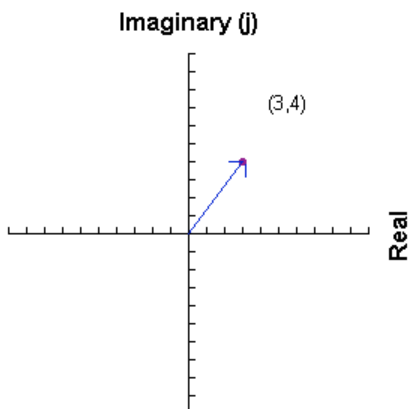
In mathematics,  $i$  is used to represent imaginary numbers. In the study of electricity and electronics,  $j$  is used to represent imaginary numbers so that there is no confusion with  $I$ , which represents current in electronics. It is also customary for scientists to write the complex number in the form  $a + jb$  (whereas in algebra we usually write it as  $a + bi$ ).

Here’s an example. Suppose the current in your AC circuit is  $I = 3 + j4$ . Now, we can visualize this complex number using the complex plane. The horizontal axis (the real axis) will keep track of the real part of the complex number. The vertical axis is the imaginary axis and will keep track of the imaginary part of the number. So this complex number corresponds to the coordinates (3, 4) on the complex plane. Here’s what it looks like:

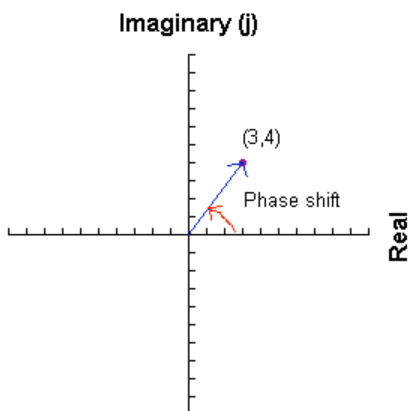


*Algebra II: Strand 3. Quadratic Functions; Topic 3. Applications; Task 3.3.3*

Now visualize drawing an arrow from the origin (0,0) to the point (3,4) (which represents the complex number  $3 + j4$ ). The length of this arrow is the size, or amplitude, of the current. The length of the arrow is the distance between (3,4) and (0,0), or  $|3 + j4|$ .



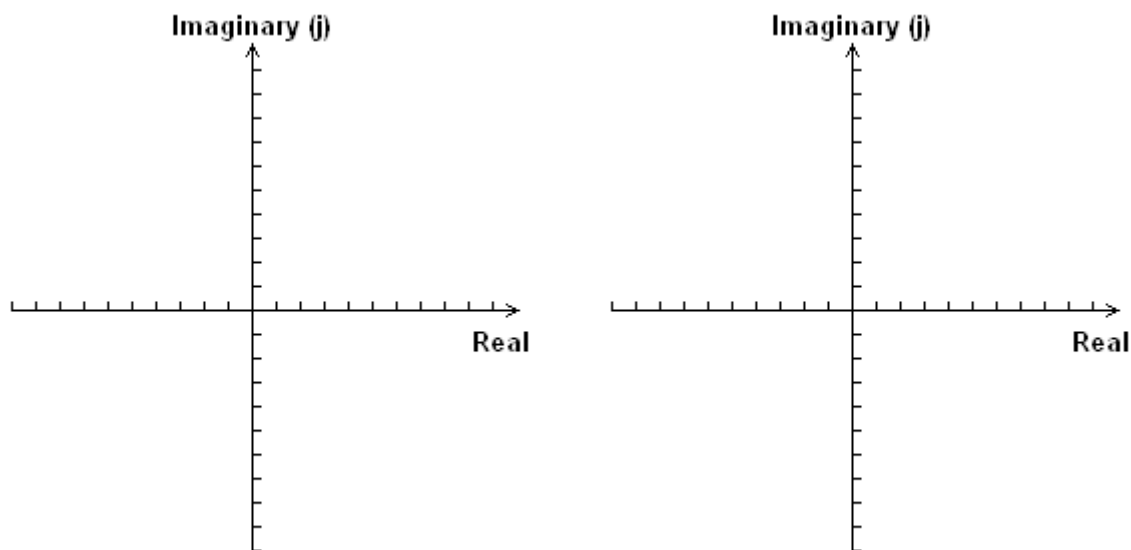
The angle that this arrow makes with the positive real axis, in the counterclockwise direction, is the phase shift.



You will need one more fact: for an AC circuit, the formula  $E = I \cdot Z$ , where **E** is **voltage**, **I** is **current**, and **Z** is **impedance**, describes the relationship between these multiple dimension quantities.

Now that you know this relationship, put on your electrical engineering hats and figure out what is happening in these AC circuits.

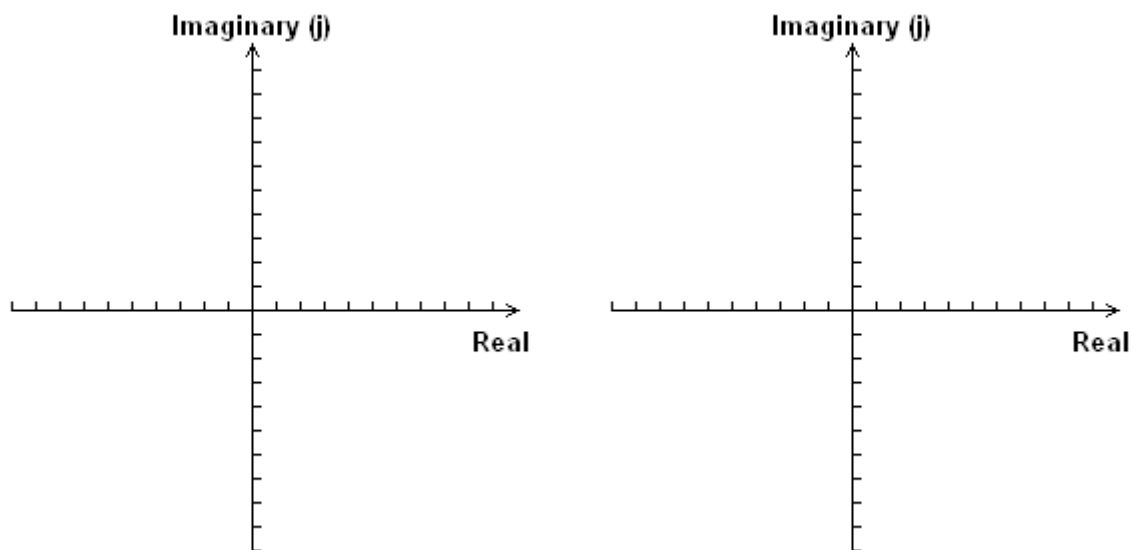
1. The impedance in *part 1* of an AC circuit is  $\mathbf{Z}_1 = 2 + j8$  ohms, and the impedance in *part 2* of the circuit is  $\mathbf{Z}_2 = 4 - j6$  ohms.
- a. Using the complex plane, plot  $\mathbf{Z}_1$  and  $\mathbf{Z}_2$ . On your graph, indicate the phase shift for each impedance.



- b. Find the magnitude of the impedance for each of  $\mathbf{Z}_1$  and  $\mathbf{Z}_2$ .
- c. Suppose the two parts of the circuit are connected in series. Find the equivalent impedance ( $\mathbf{Z}_{\text{series}}$ ) in the entire circuit, where  $\mathbf{Z}_{\text{series}} = \mathbf{Z}_1 + \mathbf{Z}_2$ .
- d. Suppose the two parts of the circuit are connected in parallel. Find the equivalent impedance ( $\mathbf{Z}_{\text{parallel}}$ ) in the entire circuit, where
- $$\mathbf{Z}_{\text{parallel}} = \frac{\mathbf{Z}_1 \cdot \mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}$$

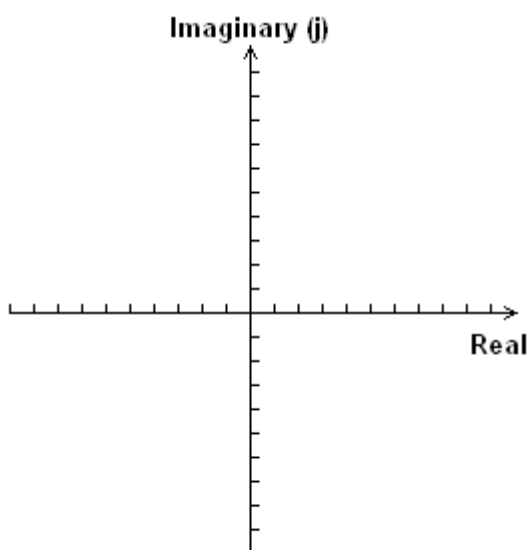
e. Find the magnitude of the impedance for each of  $Z_{\text{series}}$  and  $Z_{\text{parallel}}$ .

f. Plot  $Z_{\text{series}}$  and  $Z_{\text{parallel}}$  and indicate the phase shift.



2. The voltage in an AC circuit is  $45 + j10$  volts and the impedance is  $3 + j4$  ohms.

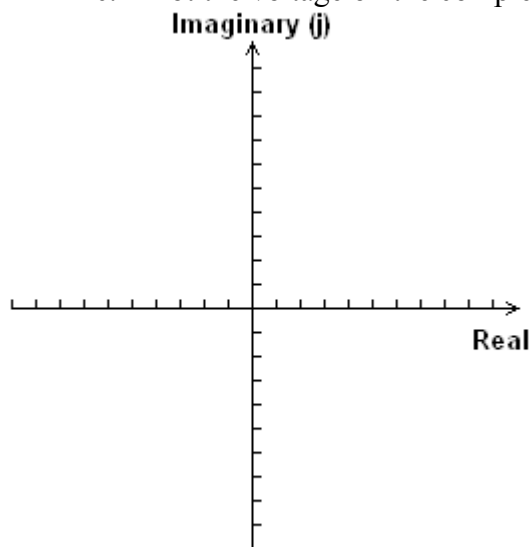
- a. What is the current?
  
  
  
  
  
  
  
  
  
  
- b. What is the amplitude of the current?
  
  
  
  
  
  
  
  
  
  
- c. Plot the current on the complex plane and indicate the phase shift.



*Algebra II: Strand 3. Quadratic Functions; Topic 3. Applications; Task 3.3.3*

3. The current in an AC circuit is  $4 + j2$  amps and the impedance is  $1 - j$  ohms.

- a. What is the voltage?
- b. What is the amplitude of the voltage?
- c. Plot the voltage on the complex plane and indicate the phase shift.



For more information on this application of complex numbers, visit these websites.

[http://www.allaboutcircuits.com/vol\\_2/chpt\\_2/1.html](http://www.allaboutcircuits.com/vol_2/chpt_2/1.html)

[http://www.st-andrews.ac.uk/~www\\_pa/Scots\\_Guide/info/signals/complex/cmplx.html](http://www.st-andrews.ac.uk/~www_pa/Scots_Guide/info/signals/complex/cmplx.html)