

TASK 3.2.1: PUTTING IT TOGETHER**Solutions**

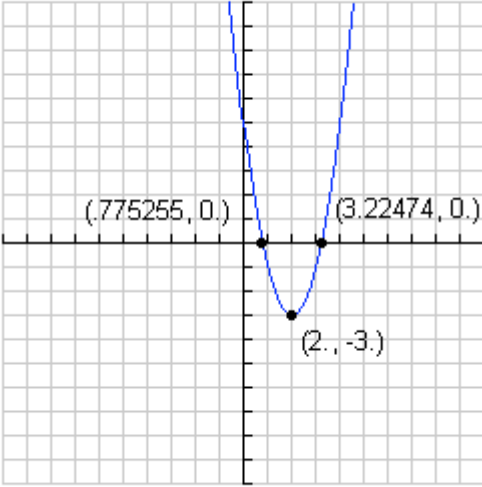
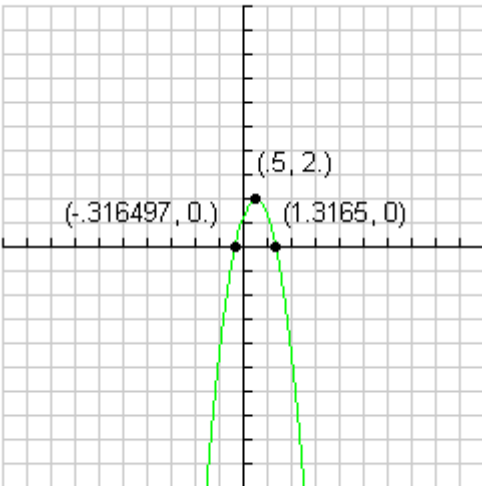
1. Each of the following quadratic functions is given in standard form ($y = ax^2 + bx + c$). For each function:
- Transform the function to the form $y = a(x - h)^2 + k$.
 - On a separate sheet of paper, sketch the graph of each equation by transforming the parent function, $y = x^2$.
 - Label the vertex of the parabola.
 - Check your graph using your calculator.
 - Use your work to complete the table below.

Function rule in $y = ax^2 + bx + c$ standard form	Function rule in $y = a(x - h)^2 + k$ vertex form	a	b	vertex
A) $y = x^2 + 2x - 1$	$y = (x + 1)^2 - 2$	1	2	(-1, -2)
B) $y = -x^2 - 6x - 8$	$y = -(x + 3)^2 + 1$	-1	-6	(-3, 1)
C) $y = 2x^2 - 8x + 5$	$y = 2(x - 2)^2 - 3$	2	-8	(2, -3)
D) $y = -3x^2 + 3x + \frac{5}{4}$	$y = -3(x - 1/2)^2 + 2$	-3	3	(1/2, 2)
E) $y = \frac{1}{2}x^2 - 2x + 1$	$y = 1/2(x - 2)^2 - 1$	1/2	-2	(2, -1)
F) $y = 3x^2 + 6x$	$y = 3(x + 1)^2 - 3$	3	6	(-1, -3)

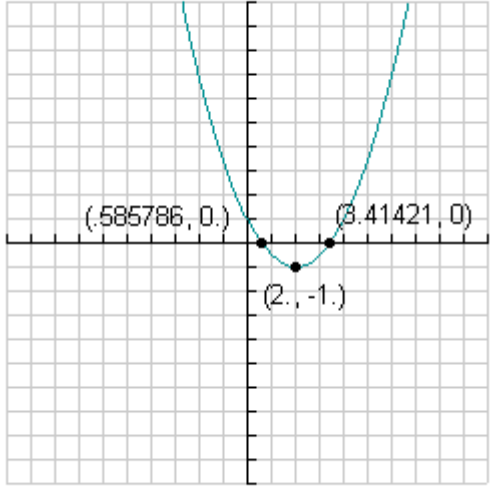
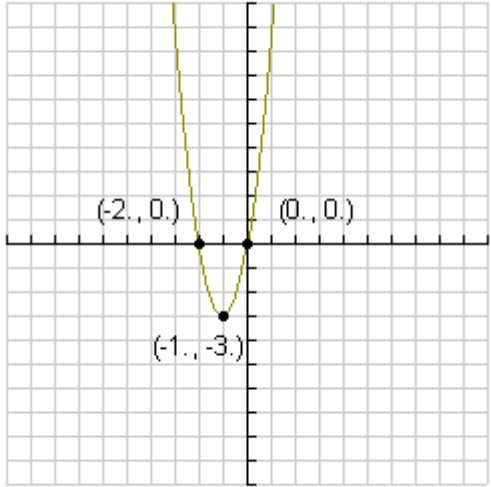
Algebra II: Strand 3. Quadratic Functions; Topic 2. Digging Deeper; Task 3.2.1

<p>A)</p>	<ul style="list-style-type: none"> • Shift left 1 • Shift down 2 	<p>Zeros: $1 \pm \sqrt{2}$</p>
<p>B)</p>	<ul style="list-style-type: none"> • Shift left 3 • Reflect across x-axis • Shift up 1 	<p>Zeros: -4, -2</p>

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<p>C)</p> 	<ul style="list-style-type: none"> • Shift right 2 • Vertical stretch by 2 • Shift down 3 	<p>Zeros:</p> $2 \pm \sqrt{\frac{3}{2}}$
<p>D)</p> 	<ul style="list-style-type: none"> • Shift right 1/2 • Reflect across x-axis • Vertical stretch by 3 • Shift up 2 	<p>Zeros:</p> $\frac{1}{2} \pm \sqrt{\frac{2}{3}}$

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<p>E)</p> 	<ul style="list-style-type: none"> • Shift right 2 • Vertical compression by $\frac{1}{2}$ • Shift down 1 	<p>Zeros: $2 \pm \sqrt{2}$</p>
<p>F)</p> 	<ul style="list-style-type: none"> • Shift left 1 • Vertical stretch by 3 • Shift down 3 	<p>Zeros: -2, 0</p>

2. Find a relationship between the x -coordinate of the vertex and the constants a and b in the standard form of the quadratic equation.

$$x \text{ coordinate} = -\frac{b}{2a}$$

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3. Find the vertex of $y = ax^2 + bx + c$ algebraically (by completing the square as you did above). Do you get the same formula for the x -coordinate of the vertex that you found by observation in question 2?

$$y = a \left(x + \frac{b}{2a} \right)^2 + c - \frac{b^2}{4a}$$

$$\text{Vertex is } \left(\frac{-b}{2a}, c - \frac{b^2}{4a} \right)$$

4. Can you use any of the work you did in Question (1) to help you find the zeros of the given quadratic functions?

Once you have the function in vertex form, it is straightforward to set the function equal to zero and solve the equation. Just add the inverse of the constant to the both sides, take positive and negative square roots, and solve for x .

5. Use your work from Question (3) to derive the quadratic formula for finding the zeros of any quadratic function in standard form, $y = ax^2 + bx + c$.

$$a \left(x + \frac{b}{2a} \right)^2 + c - \frac{b^2}{4a} = 0$$

$$\left(x + \frac{b}{2a} \right)^2 + \frac{c}{a} - \frac{b^2}{4a^2} = 0$$

$$\left(x + \frac{b}{2a} \right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$$

$$x + \frac{b}{2a} = \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

6. Now use whatever method you prefer to find the zeros of the quadratic functions A) through F) and label the zeros on the corresponding graph.

See solutions to (1) above.

Math notes

While completing the square may seem to some to be just an old-school bookkeeping algorithm, the necessity for the technique does not end with Algebra II. Students going on to Calculus will see it arise in techniques of integration, when it is important to be able to write an integrand in a particular, familiar form. It should be noted that there are multiple ways to complete the square, probably as many ways as participants in the course. This provides a good opportunity to have a discussion about the advantages and disadvantages of teaching students more than one way to do something that is multi-step and complicated.

Once participants have derived the quadratic formula, you can make a connection back to the relationship they found in question 2., where they found a representation for the x-coordinate of the vertex of the parabola, $\frac{-b}{2a}$. Note that the quadratic formula gives the x-coordinate of the zeros of the function, and they are symmetrically spaced on either side of the vertex by $\pm \frac{\sqrt{b^2 - 4ac}}{2a}$.

Teaching notes

Make certain that participants do not skip over the graphing part of the first problem. This is another opportunity to cement the connections between a formula for a function, its graph, and the transformations necessary to get that graph from the graph of the parent function.

Here is also a good chance to talk about why there is more than one form of the quadratic function. Have a discussion about when it is preferable to write a function in standard form versus vertex form and vice versa.

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E) $y = \frac{1}{2}x^2 - 2x + 1$				
F) $y = 3x^2 + 6x$				

2. Find a relationship between the x -coordinate of the vertex and the constants a and b in the standard form of the quadratic equation.
3. Find the vertex of $y = ax^2 + bx + c$ algebraically (by completing the square as you did above). Do you get the same formula for the x -coordinate of the vertex that you found by observation in question 2?

4. Can you use any of the work you did in Question 1 to help you find the zeros of the given quadratic functions?

5. Use your work from Question 3 to derive the quadratic formula for finding the zeros of any quadratic function in standard form, $y = ax^2 + bx + c$.

6. Now use whatever method you prefer to find the zeros of the quadratic functions A) through F) and label the zeros on the corresponding graph.