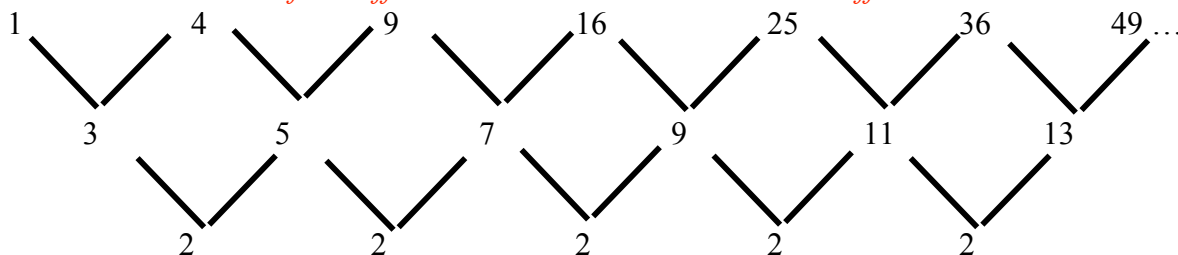


TASK 3.1.3: COUNTING DIAGONALS**Solutions**

1. Consider the pattern of perfect squares below. Find the first and second differences. What do you notice?

Notice that the first differences are linear and the second differences are constant.



2. Fill in the chart below and generate a formula for the number of diagonals of a n -sided polygon.

Allow participants time to discuss their approach to finding the quadratic formula for the number of diagonals. Point out again that the second differences are constant.

Number of sides	Figure	Number of diagonals	1 st difference	2 nd difference
3		0		
4		2	2	1
5		5	3	1
6		9	4	1
n		$\frac{n(n-3)}{2}$	$n-1$	1

- Rewrite the formula you came up with in the general form of a quadratic.

$$y = \frac{1}{2}n^2 - \frac{3}{2}n$$

- What is the relationship between the value of the second differences and the parameter a ? a is half the second differences.

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3. Find the general relationship between the first and second differences and the parameters of a quadratic equation.

Encourage participants to make a chart as in question 2 using the general quadratic equation with parameters, $y=ax^2 + bx + c$.

x	y	1 st difference	2 nd difference
0	$a(0)^2 + b(0) + c = c$		
1	$a(1)^2 + b(1) + c = a + b + c$	$a + b$	
2	$a(2)^2 + b(2) + c = 4a + 2b + c$	$3a + b$	$2a$
3	$a(3)^2 + b(3) + c = 9a + 3b + c$	$5a + b$	$2a$
4	$a(4)^2 + b(4) + c = 16a + 4b + c$	$7a + b$	$2a$
5	$a(5)^2 + b(5) + c = 25a + 5b + c$	$9a + b$	

Show how the coefficients can be obtained from this information.

c is the y -coordinate of the 0th term (the y -intercept).

a is one-half the constant common 2nd difference.

$a + b$ is the difference between the 0th and the 1st term. (Emphasize that the terms $x = 0$ and $x = 1$ must be used in order to find c and $a + b$.)

- How do these relationships connect to the answer for the diagonal question?

$$a = \frac{1}{2} \cdot 1 = \frac{1}{2}$$

Because you know the values at $x = 3$ and $x = 4$, use $7a + b$ difference.

$$7a + b = 2$$

$$b = 2 - 7a$$

$$b = 2 - \frac{7}{2}$$

$$b = -\frac{3}{2}$$

Math notes

Summarize this activity with a discussion leading towards calculus. If there are calculus teachers participating, ask them to consider connections to calculus and discuss this briefly with the group. From the time we begin talking about linear functions, we say that they have a constant rate of change. This terminology ties directly to calculus as students study the rates of change of many functions. Calculus becomes necessary to describe the rate of change of a non-linear function. In essence, differential calculus describes the variable rates of change. In this activity, we learn that quadratics have a linear rate of change. Teachers of courses prior to calculus should point this out explicitly. This may be the first time students think about a variable rate of change that we can describe. In calculus, students learn that the second derivative (the limit of the rate of change) of a quadratic function is constant. This activity sets the groundwork for that relationship using finite differences.

Teaching notes

Geometry teachers may already know a formula for the number of diagonals of a polygon, but the objective of this activity is to demonstrate how finite differences may be used to generate the formula for a quadratic function. Discuss with participants the fact that linear functions have a constant rate of change. Students often analyze this fact by making tables of values and seeing that the first difference is constant. For example, consider the linear pattern below.

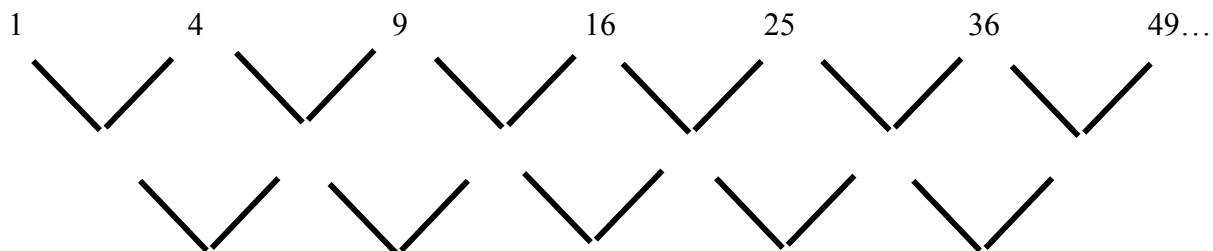
n	$3n+1$	1 st difference
0	1	
1	4	3
2	7	3
3	10	3
4	13	3

As a group, fill in the chart of second differences for the pattern of perfect squares. Point out that in this case the second differences are constant. Have the participants fill in the next two charts in small groups.

An issue that may come up is that the intervals over which we sample the quadratic functions are all unit intervals. However, the important factor here is that the intervals were all the same length. Some participants may try interval samples, say for $n = 0$, $n = 2$, $n = 3$, $n = 6$ and conclude that the differences are erratic—not constant.

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Number of sides	Figure	Number of diagonals	1 st difference	2 nd difference
3		0	2	
4		2		
5				
6				
n				

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3. Find the general relationship between the first and second differences and the parameters of a quadratic equation.

x	$y = ax^2 + bx + c$	1 st difference	2 nd difference
0			
1			
2			
3			
4			
5			