

**TASK 3.1.2: THROWING****Solutions**

The parametric equation below represents a ball being thrown straight up.

$$x(t) = 3$$

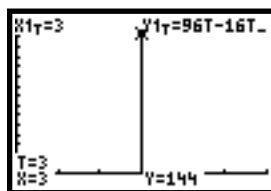
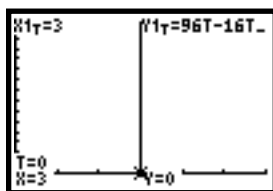
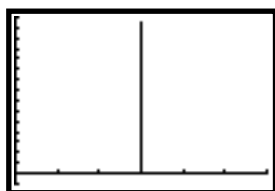
$$y(t) = 96t - 16t^2$$

1. What do you think the graph will look like? Make a sketch below.
2. Graph the parametric equation in parametric mode on your graphing calculator. Use the following window:  $t_{\min} = 0$ ,  $t_{\max} = 6$ ,  $t_{\text{step}} = .2$ ,  $x_{\min} = 0$ ,  $x_{\max} = 6$ ,  $y_{\min} = 0$ ,  $y_{\max} = 150$ . If your calculator allows, consider using a graph style that traces the path. Describe what you see on the screen.

Discuss how to set a viewing window.

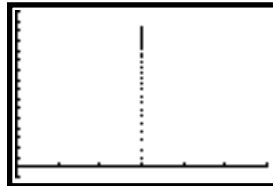
- What does  $t$  represent? *Time.*
- How might  $t$  vary? *Time should be positive values.*
- What does  $x$  represent? *In this situation,  $x$  is where the person is when the person throws the ball. We are choosing the person to be at  $x = 3$ . It is sort of a track where the ball will go up and down.*
- What does  $y$  represent? *The ball's height above the ground.*

*Below is the graph of the parametric equation. Discuss any differences that might occur between the participant's predictions and the actual graph. Make a point to discuss with the group that this graph, unlike the graph in Activity 1, actually shows the path the ball travels over time. The graph in Activity 1 graphs the height of the ball over time. The parametric equation graphs the vertical and horizontal distance of the ball. The graphs below are in the window  $t_{\min} = 0$ ,  $t_{\max} = 6$ ,  $t_{\text{step}} = 0.1$ ,  $x_{\min} = 0$ ,  $x_{\max} = 6$ ,  $y_{\min} = 0$ ,  $y_{\max} = 150$ .*



3. Trace along the path of the ball. This traces the  $t$  steps. Describe how the tracer moves along the graph.

The tracer represents the ball being thrown. It reflects the physical phenomenon of the ball slowing down as it reaches its maximum height.



Collect some data using the graph as follows. Set the tstep to one. This will give us height values every second. Trace to find the values for the following times:

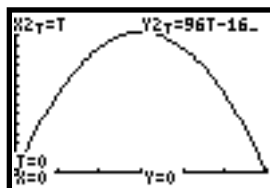
time	height
3	144
4	128
5	80
6	0

Find the first and second differences as follows:

time	height	$\Delta y$	$\Delta(\Delta y)$
3	144		
4	128	16	
5	80	48	-32
6	0	80	-32

Help participants connect that here the acceleration due to gravity shows up again as the second differences. The first differences show that the rate of change is not constant, that the rate of change is changing. The rate of change, the velocity, is changing at a constant rate, which is acceleration.

4. Sketch a graph of the height of the ball over time.



5. How can you model the above graph on your graphing calculator in parametric mode?

$$\text{Let } x(t) = t \text{ and } y(t) = 96t - 16t^2.$$

**Math notes**

This a good opportunity to use parametric equations to better illustrate an idea. Using the parametric model clarifies that you are keeping track of the height of the ball with respect to time, rather than the path of the ball.

**Teaching notes**

This activity really drives home the distinction between following the path of the ball versus following the height of the ball, while at the same time investigating a quadratic relationship. It may be necessary to illustrate a few examples that review or introduce parametric equations and their meaning before beginning this activity. This will help solidify the participants' confidence and understanding of parametric equations.

Be sure to discuss what each of the variables and the parameter represent in the situation. If the constant value of  $x$  were changed, how would that affect the height of the ball with respect to time?

Tie together the physical situation with the mathematics. Help participants connect that here the acceleration due to gravity shows up again as the second differences. The first differences show that the rate of change is not constant, that the rate of change is changing. The rate of change, the velocity, is changing at a constant rate, which is acceleration.

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