TASK 2.4.2: LINEAR PROGRAMMING APPLICATIONS

Solutions

1. A snack shop sells two kinds of mixed nuts. The standard mixture contains 100 g of cashews and 200 g of peanuts and sells for $1.85. The "Cashew Lovers" mixture contains 150 g of cashews and 100 g of peanuts and sells for $2.50. The snack shop owner has 20 kg of cashews and 30 kg of peanuts available. She would like to have at least as many standard packages as "Cashew Lovers" packages available. How many packages of each type should she make to maximize her income?

	Standard mix	*Cashew Lovers*	*Amt. Available*
Cashews	*100 g*	*150 g*	*20 kg*
Peanuts	*200 g*	*100 g*	*30 kg*
Price	*$1.85*	*$2.50*	

Let x = # of packages of standard mix and y = # of packages of cashew lovers mix.

Constraints:

$$\begin{cases} 100x + 150y \le 20{,}000 \\ 200x + 100y \le 30{,}000 \\ y \le x \\ x \ge 0 \\ y \ge 0 \end{cases}$$

Objective function: I = 1.85x + 2.50y

Intersection points of feasible region	*I=1.85x+2.50y*
(0, 0)	*I=0*
(79.998, 79.998)	*I=347.99*
(125.003, 49.993)	*I=356.24*
(150, 0)	*I=277.50*

In order to maximize the income, the snack shop owner should make 125 packages of the standard mix and 50 packages of the Cashew Lovers mix.

2. A table manufacturer has two warehouses, one in Austin and the other in Tyler. These warehouses supply stores in Ft. Worth and Houston. Every table sold at these two stores must come from one of the two warehouses. On a particular day, the Houston store gets 10 table orders and the Ft. Worth store gets 12 table orders. The Austin warehouse has 15 tables available and the Tyler warehouse has 10 tables available. The cost of shipping one table is $50 from Austin to Fort Worth, $40 from Austin to Houston, $30 from Tyler to Ft. Worth and $60 from Tyler to Houston. Create a diagram of this information. How many tables should be shipped from each warehouse to fill the orders for the day at a minimum cost?

x = # of tables shipped from Austin to Fort Worth
y = # of tables shipped from Austin to Houston
12 – x = # of tables shipped from Tyler to Fort Worth
10 – y = # of tables shipped from Tyler to Houston

$$\begin{cases} x \geq 0 \\ y \geq 0 \\ 12 - x \geq 0 \\ 10 - y > 0 \\ x + y \leq 15 \\ (12 - x) + (10 - y) \leq 10 \end{cases}$$

Objective Function: C = 50x + 40y + 30(12 – x) + 60(10 – y) = 20x – 20y + 960

Intersection points of feasible region	*C = 20x – 20y + 960*
(12, 0)	*1200*
(12, 3)	*1140*
(2, 10)	*800*
(5, 10)	*860*

In order to minimize the cost, the manufacturer should ship 2 tables from Austin to Fort Worth, 10 tables from Austin to Houston, 10 tables from Tyler to Fort Worth, and 0 tables from Tyler to Houston.

3. A nutritionist is examining the affects of various diets on lab rats. She has two types of food to choose from. Type A contains 8 grams of fat, 12 grams of carbohydrates, and 2 grams of protein per ounce. Type B contains 12 g of fat, 12 g of carbohydrates, and 1 g of protein per ounce. Type A costs $0.20 per ounce and type B costs $0.30 per ounce. Each rat needs a daily minimum of 24 g of fat, 36 g of carbohydrates, and 5 g of protein, but should get no more than 4 oz of food per day. How many ounces of each type of food should be fed to each rat daily to satisfy the dietary requirements of the experiment at a minimum cost?

Let x = # of ounces of type A and y = # of ounces of type B.
Constraints:

$$\begin{cases} x \geq 0 \\ y \geq 0 \\ x + y \leq 4 \\ 8x + 12y \geq 24 \\ 12x + 12y \geq 36 \\ 2x + y \geq 5 \end{cases}$$

The objective function is C = .20x + .30y.

Intersection points of the feasible region	***C=.20x + .30y***
(1, 3)	*$1.10*
(2, 1)	*0.70*
(3, 0)	*0.60*
(4, 0)	*0.80*

The minimum cost that meets the constraints is to give the rats 3 ounces of Type A and none of Type B.

4. A bakery makes cakes and cookies. Each cake requires preparation time, baking time, and icing time. Each process is only available for 8 hours each day. The table below gives the information about process times and expected profits from each cake and each dozen of cookies. How many cakes and dozens of cookies should be produced each day to maximize the profit?

	Cake	**Dozen cookies**	**Time available**
Preparation	5 min	12 min	8 hours
Baking	20 min	10 min	8 hours
Icing	8 min	15 min	8 hours
Expected profit	$10	$7	

Let x = # of cakes made and y = # of dozens of cookies made. Convert hours to minutes.

Constraints:

$$\begin{cases} x \geq 0 \\ y \geq 0 \\ 5x + 12y \leq 480 \\ 20x + 10y \leq 480 \\ 8x + 15y \leq 480 \end{cases}$$

The objective function is P = 10x + 7y.

Intersection points of the feasible region	***C=.20x + .30y***
(10.91, 26.18)	*$292.36*
(0, 32(	*224.00*
(24, 0)	*240.00*
(0, 0)	*0*

The maximum profit is made by making about 11 cakes and 26 dozen cookies.

Teaching notes

Assign one of the linear programming problems to each group of students. Have them create a poster of their solution including the definition of variables, the list of constraints, connections between the inequalities and the problem situation, the graph of the feasible region, the objective function and the optimal solution. Then ask them to work on the remaining problems on their own paper.

Technology notes

The TI-83plus *INEQUAL* application can be used for these problems. Refer to the technology notes for Task 1.

TASK 2.4.2: LINEAR PROGRAMMING APPLICATIONS

1. A snack shop sells two kinds of mixed nuts. The standard mixture contains 100 g of cashews and 200 g of peanuts and sells for $1.85. The "Cashew Lovers" mixture contains 150 g of cashews and 100 g of peanuts and sells for $2.50. The snack shop owner has 20 kg of cashews and 30 kg of peanuts available. She would like to have at least as many standard packages as "Cashew Lovers" packages available. How many packages of each type should she make to maximize her income?

2. A table manufacturer has two warehouses, one in Austin and the other in Tyler. These warehouses supply stores in Ft. Worth and Houston. Every table sold at these two stores must come from one of the two warehouses. On a particular day, the Houston store gets 10 table orders and the Ft. Worth store gets 12 table orders. The Austin warehouse has 15 tables available and the Tyler warehouse has 10 tables available. The cost of shipping one table is $50 from Austin to Fort Worth, $40 from Austin to Houston, $30 from Tyler to Ft. Worth and $60 from Tyler to Houston. Create a diagram of this information. How many tables should be shipped from each warehouse to fill the orders for the day at a minimum cost?

3. A nutritionist is examining the affects of various diets on lab rats. She has two types of food to choose from. Type A contains 8 grams of fat, 12 grams of carbohydrates, and 2 grams of protein per ounce. Type B contains 12 g of fat, 12 g of carbohydrates, and 1 g of protein per ounce. Type A costs $0.20 per ounce and type B costs $0.30 per ounce. Each rat needs a daily minimum of 20 g of fat, 34 g of carbohydrates, and 5 g of protein, but should get no more than 4 oz of food per day. How many ounces of each type of food should be fed to each rat daily to satisfy the dietary requirements of the experiment at a minimum cost?

4. A bakery makes cakes and cookies. Each cake requires preparation time, baking time, and icing time. Each process is only available for 8 hours each day. The table below gives the information about process times and expected profits from each cake and each dozen of cookies. How many cakes and dozens of cookies should be produced each day to maximize the profit?

	Cake	**Dozen cookies**	**Time available**
Preparation	5 min	12 min	8 hours
Baking	20 min	10 min	8 hours
Icing	8 min	15 min	8 hours
Expected profit	$10	$7	