

STRAND 2. LINEAR FUNCTIONS

TOPIC 2.3: MATRICES

Topic Notes

Mathematical focus

The mathematical focus of this section is applications of matrices. The applications will incorporate the following terms: Probability, independent event, mutually exclusive event, matrix, row, column, matrix operations

Topic overview

This topic contains five tasks:

- 2.3.1: Car Rental Model: An Application of Matrices
- 2.3.2: Does it Matter Where the Day Begins?
- 2.3.3: Transformations Using Matrices
- 2.3.4: School Fundraiser
- 2.3.5: Flying Through the St. Louis Gateway Arch

The tasks in this topic provide investigations of several applications of matrices. The applications provide review and reinforcement of concepts from probability as well as matrix operations.

TEXES standards focus

TEXES Standard I.001 Number Concepts. The teacher understands the real number system and its structure, operations, algorithms, and representations. The beginning teacher:

(E) Uses a variety of models (e.g., geometric, symbolic) to represent operations, algorithms, and real numbers.

TEXES Standard I.003 Number Concepts. The teacher understands number theory concepts and principles and uses numbers to model and solve problems in a variety of situations. The beginning teacher:

(C) Compares and contrasts properties of vectors and matrices with properties of number systems (e.g., existence of inverses, non-commutative operations).

TEXES Standard II.004 Patterns and algebra. The teacher uses patterns to model and solve problems and formulate conjectures. The beginning teacher:

(A) Recognizes and extends patterns and relationships in data presented in tables, sequences, or graphs.

TEXES Standard IV.015 Probability and statistics. The teacher understands how to use appropriate graphical and numerical techniques to explore data, characterize patterns, and describe departures from patterns. The beginning teacher:

(B) Organizes, displays, and interprets data in a variety of formats (e.g., tables, frequency distributions, scatter plots, stem-and-leaf plots, box-and-whisker plots, histograms, pie charts).

TEXES Standard IV.016 Probability and statistics. The teacher understands the concepts and applications of probability. The beginning teacher:

(F) Calculates probabilities using the axioms of probability and related theorems and concepts such as the addition rule, multiplication rule, conditional probability, and independence.

TEXES Standard V.018 Mathematical processes and perspectives. The teacher understands mathematical reasoning and problem solving. The beginning teacher:

(D) Uses formal and informal reasoning to justify mathematical ideas.

TEXES Standard V.019 Mathematical processes and perspectives. The teacher understands mathematical connections both within and outside of mathematics and how to communicate mathematical ideas and concepts. The beginning teacher:

(B) Understands how mathematics is used to model and solve problems in other disciplines (e.g., art, music, science, social science, business).

(C) Translates mathematical ideas between verbal and symbolic forms.

(D) Communicates mathematical ideas using a variety of representations (e.g., numeric, verbal, graphical, pictorial, symbolic, concrete).

(F) Uses appropriate mathematical terminology to express mathematical ideas.

TEXES Standard VI.020 Mathematical learning, instruction and assessment. The teacher understands how children learn mathematics and plans, organizes and implements instruction using knowledge of students, subject matter, and statewide curriculum (Texas Essential Knowledge and Skills [TEKS]). The beginning teacher:

(D) Understands a variety of instructional strategies and tasks that promote students' abilities to do the mathematics described in the TEKS.

TEKS/TAKS focus

TEKS 2A.2 Foundations for functions. The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations. The student is expected to:

(A) use tools including factoring and properties of exponents to simplify expressions and to transform and solve equations.

TEKS 2A.3 Foundations for functions. The student formulates systems of equations and inequalities from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situations. The student is expected to:

(A) analyze situations and formulate systems of equations in two or more unknowns or inequalities in two unknowns to solve problems; and

(B) use algebraic methods, graphs, tables, or matrices, to solve systems of equations or inequalities.

TEKS 2A.6 Quadratic and square root functions. The student understands that quadratic functions can be represented in different ways and translates among their various representations. The student is expected to:

(C) determine a quadratic function from its roots or a graph.

TEKS 2A.7 Quadratic and square root functions. The student interprets and describes the effects of changes in the parameters of quadratic functions in applied and mathematical situations. The student is expected to:

(A) use characteristics of the quadratic parent function to sketch the related graphs and connect between the $y = ax^2 + bx + c$ and the $y = a(x - h)^2 + k$ symbolic representations of quadratic functions; and

(B) use the parent function to investigate, describe, and predict the effects of changes in a , h , and k on the graphs of $y = a(x - h)^2 + k$ form of a function in applied and purely mathematical situations.

TEKS 2A.8 Quadratic and square root functions. The student formulates equations and inequalities based on quadratic functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. The student is expected to:

(D) solve quadratic equations and inequalities using graphs, tables, and algebraic methods.

High School TAKS Objective 1: The student will describe functional relationships in a variety of ways.

High School TAKS Objective 2: The student will demonstrate an understanding of the properties and attributes of functions.

High School TAKS Objective 4: The student will formulate and use linear equations and inequalities.

High School TAKS Objective 5: The student will demonstrate an understanding of quadratic and other nonlinear functions.

High School TAKS Objective 10: The student will demonstrate an understanding of mathematical processes and tools used in problem solving.

Materials

Materials needed	Task 1a,b	Task 2	Task 3	Task 4	Task 5
Graphing calculator	*	*	*	*	*
MATPLOt program			*		
Graph paper			*		

Procedure

Have a discussion with the group about what matrices are and how they currently teach matrices. Solicit ideas for how matrices are used in the real world. This should be a very informal discussion intended to get the participants thinking about matrices. Look at task 2.3.4: School Fundraiser for an intuitive way to introduce matrices and matrix operations to students.

Go through the set up of the problem with the participants. Make sure that everyone understands the chart that is given. Walk through question 1 as a large group then ask participants to work in small groups on question 2. Walk through question 3 as a large group, then let the small groups work on question 4. There are bulleted questions to help guide the participants through the remainder of the activity.

Tasks 2.3.1a, 2.3.1b and 2.3.2 are an application of matrices related to Markov Chains. The situation is presented in Task 2.3.1a, the matrix is developed and applied in Task 2.3.1b and extended in Task 2.3.2.

Task 2.3.3 focuses on using matrices to represent transformations. When participants determine the general rotation matrix, a rich discussion can be held to link the matrix multiplication to compositions of transformations.

Tasks 2.3.4 and 2.3.5 are suitable for student use and provide participants with approaches to working with matrices in the context of applications that is not generally done in traditional high school mathematics courses.

Summary

These activities allow participants to use matrices to deal with data. The first two tasks look at a probability process known as a Markov Chain. In the third task, we see how the data from transformations on a coordinate grid is organized with matrices. Matrices are a very useful tool in applied mathematics, and we see the power of using matrices in these activities.

Assessments

Ask participants to complete a Teacher Journal for the task 2.3.2 and one Student Journal for the tasks 2.3.3-2.3.5. Ask participants to create a matrix problem solving situation from a textbook problem.

	Teacher use only	Modify for students	Ready for students
Task 1a,b, 2		*	
Task 3			*
Task 4			*
Task 5			*

Additional notes

Reference: Application to Markov Chains: Don't you like to see the future sometimes? <http://aix1.uottawa.ca/~jkhoury/markov.htm> Downloaded 3/23/2004.