

TASK 2.3.4: SCHOOL FUNDRAISER**Solutions**

Suppose you are analyzing four school wide fundraisers at Sunset High School. Each grade level reported their sales to their class sponsor. The fundraisers consisted of candles, wrapping paper, gift bags and chocolate bars. The table below shows the total number of items sold in each class.

	Candles	Wrapping paper	Gift bags	Chocolate bars
Freshman (F)	400	550	148	742
Sophomore (SP)	275	450	200	800
Junior (J)	520	400	115	635
Senior (SR)	700	630	270	675

Instead of organizing the data in a table as above, we could put the data into a matrix as shown below:

$$S = \begin{bmatrix} 400 & 550 & 148 & 742 \\ 275 & 450 & 200 & 800 \\ 520 & 400 & 115 & 635 \\ 700 & 630 & 270 & 675 \end{bmatrix}$$

The matrix S is a 4x4 matrix where the rows show each class's results and the columns show the results for each type of item.

- Suppose another school in the district, Tyler High School, participated in the same fundraisers. The number of items sold on their campus is organized in matrix T using the same categories.

$$T = \begin{bmatrix} 240 & 370 & 90 & 345 \\ 180 & 275 & 110 & 235 \\ 330 & 260 & 283 & 328 \\ 239 & 315 & 88 & 240 \end{bmatrix}$$

Explain the meanings of each entry in matrix T. **The freshman class sold 240 candles, 370 rolls of wrapping paper, 90 gift bags and 345 bars of chocolate. The sophomore class sold 180 candles, 275 rolls of wrapping paper, 110 gift bags and 235 bars of chocolate. The junior class sold 330 candles, 260 rolls of wrapping paper, 283 gift bags and 328 bars of chocolate. The senior class sold 239 candles, 315 rolls of wrapping paper, 88 gift bags and 240 bars of chocolate.**

Determine the total amount of sales for each item and class in both schools.
Add each element of matrix S to the corresponding element of matrix T.

Algebra II: Strand 2. Linear Functions; Topic 3. Matrices; Task 2.3.4

$$S+T = \begin{bmatrix} 640 & 920 & 238 & 1087 \\ 455 & 725 & 310 & 1035 \\ 850 & 660 & 398 & 963 \\ 939 & 945 & 358 & 915 \end{bmatrix}$$

What matrix operation are you performing when you calculate the data in the question above?

Matrix addition

2. Let's introduce another matrix, P that represents the price of each of the items sold in the fundraisers. Suppose the candles sold for \$15 each, the wrapping paper sold for \$7 per roll, the gift bags sold for \$5 each and the chocolate bars sold for \$2 each. Create a 4x1 matrix, P, that represents this information.

$$P = \begin{bmatrix} 15 \\ 7 \\ 5 \\ 2 \end{bmatrix}$$

3. Determine the amount of money raised by each class in the Sunset High School (matrix S). Explain your process.
- If we want to organize our answer in a matrix, what dimensions will the matrix have? 4x1
 - What will the entries in the matrix represent? Each row will represent the class and the number will represent the total amount of money raised by that class on all four products.
 - How much money did the freshman class make on candles?
In order to determine the amount of money the freshman class raised on candles we would multiply 400 (candles sold) by \$15 (price per candle).
 $400 \cdot \$15 = \$6,000$
 - How much money did the freshman class make on wrapping paper? $550 \cdot \$7 = \$3,850$
 - How much money did the freshman class make on gift bags?
 $148 \cdot \$5 = \740
 - How much money did the freshman class make on chocolate bars?
 $742 \cdot \$2 = \$1,484$
 - What value should we enter into the first row of our answer matrix?
Where does this number come from? We need to find the sum of amounts collected for each individual product. $\$6000 + \$3850 + \$740 + \$1484 = \$12,074$
 - Notice where these number came from in each matrix. Use the matrix to find the total amount raised by the sophomore class. Explain where the values came from in the matrices. Multiply each row element in matrix S

by each column element in matrix P and find the sum.

$(275)(15)+(450)(7)+(200)(5)+(800)(2)=9,875$. So the sophomore class raised \$9,875.

- Find the remaining two entries for our answer matrix. 12,445 and 17,610.

$$R = \begin{bmatrix} 12074 \\ 9875 \\ 12445 \\ 17610 \end{bmatrix}$$

So,

- Write a matrix equation that represents this problem. Use your calculator to check your solution.

$S \cdot P = R$

$$\begin{bmatrix} 400 & 550 & 148 & 742 \\ 275 & 450 & 200 & 800 \\ 520 & 400 & 115 & 635 \\ 700 & 630 & 270 & 675 \end{bmatrix} \cdot \begin{bmatrix} 15 \\ 7 \\ 5 \\ 2 \end{bmatrix} = \begin{bmatrix} 12074 \\ 9875 \\ 12445 \\ 17610 \end{bmatrix}$$

- How would we have to write the above equation if we did not use matrices?

$(400)(15)+(550)(7)+(148)(5)+(742)(2)=12,074$

$(275)(15)+(450)(7)+(200)(5)+(800)(2)=9,875$

$(520)(15)+(400)(7)+(115)(5)+(635)(2)=12,445$

$(700)(15)+(630)(7)+(270)(5)+(675)(2)=17,610$

4. Use your result matrix from above to determine the total amount of money raised by the school on these fundraisers.

Find the sum of all the elements of matrix R. The total amount raised by the school is \$52,004.

- Is this a matrix operation? Why or why not? No, we are not performing an operation on two matrices. We are manipulating the elements of one matrix.

Calculator note: Most calculators can find the sum above in a couple of ways. There is a matrix operation that finds the cumulative sum of the entries in a matrix and lists them as a matrix. So in this case it would result in another 4×1 matrix where the entry in row 1 stays the same, the entry in row 2 is the sum of row 1 and 2, the entry in row 3 is the sum of rows 1, 2 and 3 and the entry in row 4 is the sum of rows 1, 2, 3, and 4. Thus, the entry in row 4 is the answer to our question. Another way to find this sum is to save the matrix as a list and find the sum of the list.

5. Tyler High School, whose sales are described in matrix T, presented the results from their fundraisers at a community meeting. The total amount of money raised by each class are given by matrix V below.

$$V = \begin{bmatrix} 7465 \\ 5805 \\ 8586 \\ 6786 \end{bmatrix}$$

Did they use the same price matrix as Sunset High School? How do you know? If not, what price did they sell each item for?

We know they did not use the same price matrix, because with the graphing calculator we can easily compute $T \cdot P$ and see that it does not equal V . So, we have to solve the following matrix equation for C .

$$T \cdot C = V$$

$$\begin{bmatrix} 240 & 370 & 90 & 345 \\ 180 & 275 & 110 & 235 \\ 330 & 260 & 283 & 328 \\ 239 & 315 & 88 & 240 \end{bmatrix} \cdot \begin{bmatrix} c \\ w \\ g \\ b \end{bmatrix} = \begin{bmatrix} 7465 \\ 5805 \\ 8586 \\ 6786 \end{bmatrix}$$

- Algebraically, how would we solve for C in the equation $T \cdot C = V$? We could multiply both sides of the equation by the inverse of T .
- What is different about matrix multiplication than numerical multiplication that will affect the solution to this equation? Matrix multiplication is not commutative. So, if we want to multiply both sides of the equation to get C alone, we must multiply the inverse of T to the left of T . This also requires that we multiply T^{-1} to the left of V . So our solution equation would be $T^{-1} \cdot T \cdot C = T^{-1} \cdot V$. Since finding the inverse of a 4×4 matrix is very cumbersome, we can use a calculator with matrix functions to solve this equation.
- Do all matrices have an inverse? No, only square matrices have an inverse.
- What should the product of a matrix and its inverse yield? By definition, it gives us the identity matrix.
- What is the identity matrix for a 4×4 matrix?

$$I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Use your calculator to verify that $T^{-1} \cdot T$ does in fact give you the identity matrix.

$$C = \begin{bmatrix} 12 \\ 10 \\ 6 \\ 1 \end{bmatrix}$$

The price matrix used by the second school is

Math notes

Students use matrices and matrix operations from an intuitive approach to organize information.

Teaching notes

Ask students to work in small groups on the activity sheet. Present the situation and discuss the use of a matrix to organize the data in the table. This activity is a good example of calculations that can be done without matrices but are very cumbersome. It is good motivation for using matrices. Let students grapple with the large number of calculations by hand to get an intuitive sense of what is going on. Then use matrix operations to manage the data. Questions 1-4 present an intuitive approach to matrix operations. This would be a good breaking point for students to have some practices with smaller matrices. Then come back to the remaining questions, which deal with solving systems of equations with matrices.

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3. Determine the amount of money raised by each class in Sunset High School (matrix S). Explain your process.
4. Use your result matrix from above to determine the total amount of money raised by the school on these fundraisers.

5. Tyler High School, whose sales are described in matrix T , presented the results from their fundraisers at a community meeting. The total amounts raised by each class are given by matrix V below.

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