STRAND 2: LINEAR FUNCTIONS

TOPIC 2.2: SLOPE AND RATE OF CHANGE

Topic Notes

Mathematical focus

The mathematical focus of this topic is to develop the concept of slope as a rate of change via examination of average rate of change. Also, emphasized is that a distinguishing characteristic of all linear functions is that they have a constant rate of change.

Topic overview

This topic contains two tasks:

2.2.1: Average Rates of Change
2.2.2: What Makes a Function Linear?

Participants will explore the average rate of change of three linear functions, one quadratic function, one cubic function, and one exponential function. Graphing calculator technology will be used to generate a table of function values for each function. Average rates of change will be calculated for each function by means of calculating successive differences using the list features of the graphing calculator. The tasks in this topic lead participants toward building understanding that linear functions always have a constant average rate of change (equivalent to their slope) over any interval and that there are multiple ways to represent and describe linear functions via verbal, graphic, tabular, and symbolic representations.

TExES standards focus

TExES Standard II.004 Patterns and algebra. The teacher uses patterns to model and solve problems and formulate conjectures. The beginning teacher:

(A) Recognizes and extends patterns and relationships in data presented in tables, sequences, or graphs.
(B) Uses methods of recursion and iteration to model and solve problems.

TExES Standard II.006 Patterns and algebra. The teacher understands linear and quadratic functions, analyzes their algebraic and graphical properties, and uses them to model and solve problems.The beginning teacher:

(A) Understands the concept of slope as a rate of change and interprets the meaning of slope and intercept in a variety of situations.

TExES Standard II.007 Patterns and algebra. The teacher understands polynomial, rational, radical, absolute value, and piecewise functions, analyzes their algebraic and graphical properties, and uses them to model and solve problems. The beginning teacher:

(A) Recognizes and translates among various representations (e.g., written, tabular, graphical, algebraic) of polynomial, rational, radical, absolute value, and piecewise functions.

TEKS/TAKS focus

TEKS Alg II (a) (3) Basic understandings.
Functions, equations, and their relationship. The study of functions, equations, and their relationship is central to all of mathematics. Students perceive functions and equations as means for analyzing and understanding a broad variety of relationships and as a useful tool for expressing generalizations.

TEKS 2A.2 Foundations for functions. The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.
(C) student connects the function notation of y = and f(x) =.

High School TAKS Objective 2: The student will demonstrate an understanding of the properties and attributes of functions.

High School TAKS Objective 3: The student will demonstrate an understanding of linear functions.

Materials

Materials Needed	Task 2.2.1	Task 2.2.2
Graphing calculator	√	√
Flip chart with ruled squares		√
Colored markers		√
Masking tape		√
Meter sticks (straightedge)		√

Procedure

Begin by reviewing what it means for a function to be increasing on an interval, decreasing on an interval, or constant on an interval. Call participants' attention to the definitions given on pages1-3 of Task 2.2.1. Point out that functions can be increasing on an interval, decreasing on a different interval, and constant on another. When discussing the definitions on pages 1-3, ask participants for input in deciding over which intervals the functions presented are increasing, decreasing, or constant.

Discuss the importance of the sign of the average rate of change function if we know that a function is increasing, decreasing, or constant on an interval. Also, point out that the magnitude of the average rate of change of f from c to d gives us an idea about how fast (or slow) a function is changing, on the average, on an interval (c,d).

Ask participants to work individually to complete Task 2.2.1. If they are unfamiliar with the graphing calculator and, in particular, using the statistics and list features, then you may need to work through an example with them. Refer to the Teacher notes for Task 2.2.1 for calculator instructions and screen shots.

After they complete Task 2.2.1, ask them to form pairs or triads for Task 2.2.2. Ask them share their results from Task 2.2.1 among their group and discuss observations about the differences between linear functions and nonlinear functions. Then, give them Task 2.2.2. In this task, they are to answer the question, "What makes a function linear?" They are to answer the question, based on their results from Task 2.2.1, and are to also include an example of a linear function represented verbally, graphically, in tabular form, and symbolically. Ask them to record their responses on poster paper and post work around the room. Allow a few minutes for a gallery walk to allow each group to view the work of the other groups. Conclude with a brief discussion of various ways to verbally represent linear functions.

Summary

Participants should come away from these tasks with the understanding that what distinguishes a linear function is that it has a constant rate of change. They should be able to find the average rate of change of any function between two given points. They should understand and be able to prove that the rate of change of a linear function is the slope. They should also recognize that linear functions can be represented in verbal, graphic, tabular, and symbolic forms. They should be able to give examples of each representation and be able to indicate how the constant rate of change appears in each representation.

Extensions

Point out that linear functions in Problems 1 through 3 were chosen to have Δx = 1 to make the calculations easy. If the Δx value is other than 1, which it often is, then it is necessary to find the value of Δx as well as the value of $\Delta f(x)$ in order to find the rate of change.

Note that this difference method can be used to identify functions other than linear. If the x values are uniformly spaced, then constant second differences indicate a quadratic. Ask participants to take another look at Problem 4. Ask them to find the differences $\dfrac{r_{x_i}\left(x_{i+1}\right)-r_{x_{i+1}}\left(x_{i+2}\right)}{x_{i+1}-x_i}$ (second differences). There will be a constant difference. Ask them to find successive quotients of $\dfrac{r_{x_i}\left(x_{i+1}\right)}{f(x_i)}$ in

Problem 6. The result will be a constant quotient. This will be true of exponentials if the x values are uniformly spaced.

Assessments

Task 2.2.2. may be used as an assessment.