TASK 2.1.2: FINITE DIFFERENCES

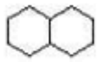
1. Refer to your charts from Task 2.1.1 and add two columns to each chart:
   a. A column on the left of the chart labeled "Change in the number of cars: Δn"
   b. A column on the right side of the chart labeled "Change in Perimeter: Δp"

   Examine each chart, what connections are there between the ratio $\frac{\Delta p}{\Delta n}$ and your findings in Task 2.1.1? Explain your reasoning.

Changes in # of cars Δn	Position in Sequence Number of cars in train (each square represents 1 train car)	Process	Term Value Perimeter of Train	Change of Perimeter Δp
	□	Front + back + 1 car with (top + bottom) 2 + 1(2)	4	
1				2
	2 □□	Front + back + 2 cars with (top + bottom) 2 + 2(2)	6	
1				2
	3 □□□ Draw Train	Front + back + 3 cars with (top + bottom) 2 + 3(2)	8	
1				2
	4 □□□□ Draw Train	2 + 4(2)	10	
	…			
	15 Predict	Front + back + 15 cars with (top + bottom) 2 + 15(2)	32	
	n	Stays the same + (# of cars in train)(rate) 2 + n(2) = 2 + 2n	2+2n	

Change in # of cars n	Position in Sequence Number of cars in train	Process	Term Value Perimeter of Train	Change in Perimeter P
	1	Front + back + 1 horizontal 2 + 1	3	
1				1
	2	Front + back + 2 horizontal 2 + 2	4	
1				1
	3 Draw Train	Front + back + 3 horizontal 2 + 3	5	
1				1
	4 Draw Train	Stays the same + (what changes) 2 + (4)	6	
	...			
	15 Predict	2 + (15)	17	
	n	2+2*n*	2+2n	

Changes in # of cars Δn	Position Sequence Number of cars in Train	Process	Term Value Perimeter of Train	Changes in Perimeter P
	1	(Front + back) + (bottom with length 2 + top with length 1) · 1 car 2 + 3 (1)	5	
1				3
	2	(Front + back) + (3 bottom and top segments) · 2 cars 2 + 3(2)	8	
1				3
	3 Draw Train	2+3(3)	11	
1				3
	4 Draw Train	2+3(4)	14	
	...			
	15	2+3(15)	47	
	n	(Front + back) + (3 bottom and top segments) · n cars 2+3*n*	2+3*n*	

Change in # of cars Δn	Position Sequence Number of cars in train	Process	Term Value Perimeter of train	Change in Perimeter Δp
	1 	(Front + back) + (2 top + 2 bottom) · 1 car 2 + 4 (1)	6	
1	2 	(Front + back) + (2 top + 2 bottom) · 2 cars 2 + 4 (2)	10	4
1	3 Draw Train	(Front + back) + (2 top + 2 bottom) · 3 cars 2 + 4 (3)	14	4
1	4	(Front + back) + (2 top + 2 bottom) · 4 cars 2 + 4 (4)	18	4
	…	Stays the same + rate of change (what changes)		
	15 Predict	(Front + back) + (2 top + 2 bottom) · 15 cars 2 + 4 (15)	62	
	n	2 + 4n	2 + 4n	

2. Why is it important to consider the ratio $\frac{\Delta p}{\Delta n}$ rather than simply comparing Δp (the rate) for each change in n when determining whether or not there is a linear pattern?
   Many times finite difference examples are misleading in that they typically illustrate finite differences over intervals of equal length (in most cases length=1 unit). Thus, participants may incorrectly assume that if they were to calculate finite differences for, say, n=1, 3, 4, 7, 9, for a linear function—the change in p would not be constant—that because the finite differences were not constant, the function is not linear. Here we hope to see answers that illustrate that although the change in p is not constant they would have to check the ratio $\frac{\Delta p}{\Delta n}$ before making any conclusions.

Math notes
Investigating finite differences is a typical task for middle school students. The focus of this task is in underscoring the importance of looking at the ratio of the differences rather than only the resulting difference in the function values without regard to the change in the independent variable.

Teaching notes
This task may be assigned as homework after the instructor illustrates how to expand the charts (adding the extra columns) and demonstrates finding the differences for a few rows. A class discussion should take place at the beginning of the next session where

participants share their results in groups. Ensure that the idea that the graph of any function that produces a constant ratio of finite differences must be a line or that the function must be linear.

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   Examine each chart. What connections are there between the ratio $\frac{\Delta p}{\Delta n}$ and your findings in Task 2.1.1? Explain your reasoning.

2. Why is it important to consider the ratio $\frac{\Delta p}{\Delta n}$ rather than simply comparing Δp (the rate) for each change in n when determining whether or not there is a linear pattern? Explain and give an example that clarifies your explanation.

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