

STRAND 1: FOUNDATIONS OF FUNCTIONS**TOPIC 1.3: APPLYING TRANSFORMATIONS****Topic 1.3 Notes****Mathematical focus**

For this topic, the mathematical focus is two-fold. First, participants must use their knowledge of transformations and key features of functions in order to use technology effectively to examine function behavior. Secondly, participants must apply their knowledge of transformations to solve application problems.

Topic overview

There are seven tasks included in this topic.

Task 1.3.1: Good Viewing Windows

Task 1.3.2: Friendly Viewing Windows

Task 1.3.3: Reflect and Apply

Task 1.3.4: Matching Parent Functions

Task 1.3.5: Tree Growth (Part I, II, III)

Task 1.3.6: Two Transformations

Task 1.3.7: Describing Transformations

Participants use their knowledge of transformations to determine appropriate viewing windows on graphing calculators to examine new functions created from parent functions. Participants use real data to graph a scatter plot and find an equation that models a trend. They use transformations to explain aspects of the trend. Participants find a real application for horizontal shifts.

TEXES standards focus

TEXES Standard II.004 Patterns and algebra. The teacher uses patterns to model and solve problems and formulate conjectures. The beginning teacher:

(A) Recognizes and extends patterns and relationships in data presented in tables, sequences, or graphs.

TEXES Standard II.005 Patterns and algebra. The teacher understands attributes of functions, relations, and their graphs. The beginning teacher:

(B) Identifies the mathematical domain and range of functions and relations and determines reasonable domains for given situations.

(E) Applies basic transformations [e.g., $k f(x)$, $f(x) + k$, $f(x - k)$, $f(kx)$, $|f(x)|$] to a parent function, f , and describes the effects on the graph of $y = f(x)$.

(F) Performs operations (e.g., sum, difference, composition) on functions, finds inverse relations, and describes results symbolically and graphically.

TEExES Standard II.006 Patterns and algebra. The teacher understands linear and quadratic functions, analyzes their algebraic and graphical properties, and uses them to model and solve problems. II.006 F, G

(F) Solves problems involving quadratic functions using a variety of methods (e.g., factoring, completing the square, using the quadratic formula, using a graphing calculator).

(G) Models and solves problems involving linear and quadratic equations and inequalities using a variety of methods, including technology.

TEExES Standard V.019 Mathematical processes and perspectives. The teacher understands mathematical connections both within and outside of mathematics and how to communicate mathematical ideas and concepts.

(C) Translates mathematical ideas between verbal and symbolic forms.

(D) Communicates mathematical ideas using a variety of representations (e.g., numeric, verbal, graphical, pictorial, symbolic, concrete).

(E) Understands the use of visual media, such as graphs, tables, diagrams, and animations, to communicate mathematical information.

TEKS/TAKS focus

TEKS 2A.1 Foundations of functions. The student uses properties and attributes of functions and applies functions to problem situations. The student is expected to:

(A) identify the mathematical domains and ranges of functions and determine reasonable domain and range values for continuous and discrete situations; and

(B) collect and organize data, make and interpret scatter plots, fit the graph of a function to the data, interpret the results, and proceed to model, predict, and make decisions and critical judgments.

TEKS 2A.4 Algebra and geometry. The student connects algebraic and geometric representations of function. The student is expected to:

(A) identify and sketch graphs of parent functions, including linear ($f(x) = x$), quadratic ($f(x) = x^2$), exponential ($f(x) = a^x$), and logarithmic ($f(x) = \log_a x$) functions, absolute value of x ($f(x) = |x|$), square root of x ($f(x) = \sqrt{x}$), and reciprocal of x ($f(x) = 1/x$); and

(B) extend parent functions with parameters such as a in $f(x) = a/x$ and describe the effects of the parameter changes on the graph of parent functions.

High School TAKS Objective 1: The student will describe functional relationships in a variety of ways.

High School TAKS Objective 2: The student will demonstrate an understanding of the properties and attributes of functions.

High School TAKS Objective 3: The student will demonstrate an understanding of linear functions.

High School TAKS Objective 5: The student will demonstrate an understanding of quadratic and other nonlinear functions

High School TAKS Objective 9: The student will demonstrate an understanding of percents, proportional relationships, probability, and statistics in application problems.

High School TAKS Objective 10: The student will demonstrate an understanding of the mathematical processes and tools used in problem solving.

Materials

| | Task 1 | Task 2 | Task 3 | Task 4 | Task 5 | Task 6 | Task 7 |
|---|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
| Graphing Calculator | x | x | x | | x | x | |
| Calculator Overhead Viewing Screen | x | | | | | | |
| Colored paper | | | | x | | | |

Procedure

These tasks are meant to give participants a feel for applying transformations to real-life situations. The tasks are not meant to find exact models using regression. When presenting these tasks, emphasize application of transformations. Task 1.3.5 (I, II, III) may be worked in class and Task 1.3.6 and 1.3.7 assigned as homework.

Summary

The primary focus of the first part of these tasks is to emphasize that a “good viewing window” will show the important features of a graph. Also, that a “friendly viewing window” shows a non-distorted shape of the graph and admits better values for tracing. These tasks provide a review of the basic characteristics of parent functions and lead to the applications tasks based upon modeling data.

Assessments

- Task 1.3.3 can be completed as an assessment after Tasks 1.3.1 and 1.3.2.
- Task 1.3.7 can be completed as an assessment after Tasks 1.3.5 and 1.3.6.
- Have participants create an assessment of their own and present it to their group or the class.
- Have participants complete one teacher journal after completing Task 1.3.5 (I, II, III).
- Have participants complete a student journal after completing Tasks 1.3.6 and 1.3.7.

Transitions to the classroom—which of the tasks are appropriate only for teachers? Which for students? How would we modify a teacher-task to make it student appropriate?

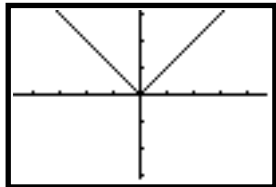
| | Teacher use only | Modify for students | Ready for students |
|-------------------|-------------------------|----------------------------|---------------------------|
| Task 1.3.1 | | x | |
| Task 1.3.2 | x | | |
| Task 1.3.3 | | x | |
| Task 1.3.4 | | | x |
| Task 1.3.5 | | x | |
| Task 1.3.6 | | x Extension | x Task |
| Task 1.3.7 | | | x |

Assessment

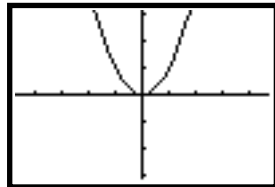
1. Match the functions with the graphs and write the domain and range under each graph.

_____ a. $y = x^3$

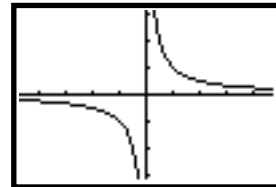
1.



2.



3.

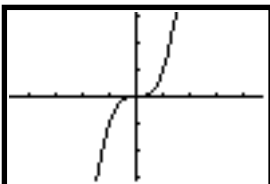


_____ b. $y = 2^x$

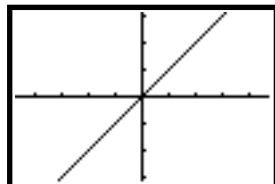
_____ c. $y = |x|$

_____ d. $y = \ln x$

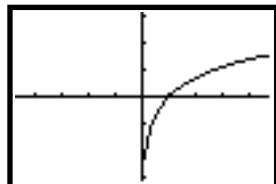
4.



5.



6.

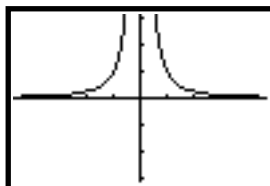


_____ e. $y = \sqrt{x}$

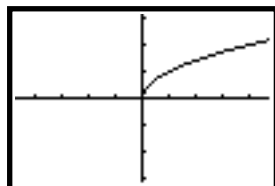
_____ f. $y = x$

_____ g. $y = \frac{1}{x}$

7.



8.



9.



_____ h. $y = x^2$

_____ i. $y = \frac{1}{x^2}$