Task 1.3.6: TWO TRANSFORMATIONS

Solutions

The Livestock Show and Rodeo School Art Program is an annual competition of the city's students. Participants are in grades ranging fro kindergarten through 12 and must submit an original art project based on Western culture, history, or heritage. Projects are generally created in the fall and then judged by qualified individuals from the show's School Art Committee. Individual school districts select the top 20 students to compete in this annual citywide competition.

The scores for the top Rodeo School Art Competition entries in the high school division in East District are listed below.

Entry	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Score	65	71	74	77	79	80	81	82	83	84	85	86	87	89	90	90	91	92	93	95

The Art Committee guidelines state that the top rating awarded in district competitions should be 100. The East District judges have decided to add 5 points to each score in order to comply with the competition guidelines.

1. Create a table to show the new scores. Compare the mean and median of the original scores with those of the modified scores.

The table below was created using a graphing calculator. The first list is the entry number. The second list is a given score the third list is created by adding 5 to the second list value.

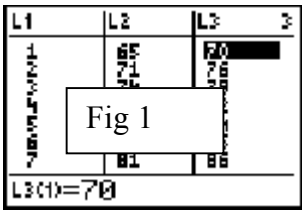


Fig 1

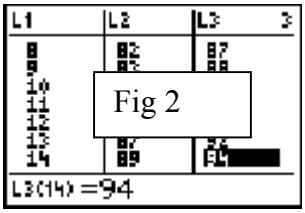


Fig 2

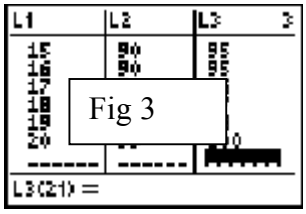


Fig 3

Finding the sum of the original scores and dividing by 20 yields the mean. The sum of the original score is 1,674, and the mean is 83.7. The sum of the modified scores is 1,774, and the mean is 88.7. The mean of the modified score is 5 points higher than the mean of the original scores.

Because the scores are printed in numerical order, the median is the average of the tenth and eleventh scores in the list. The median of the original scores is the average of 84 and 85 or 84.5. The median of the modified scores is the average of the tenth and eleventh scores on the modified list. The average of 89 and 90 is 89.5. The median of the set of modified scores is 5 points higher that the median of the original set of scores.

2. The judges want to see a visual representation of the scores. Graph the original scores using the entry number as the x-coordinate and the score as the y-coordinate. Describe the parent function to which this graph belongs.

   Ask the question: What would be a good viewing window to see the scatter plot? A possible window and the scatterplot are shown below.

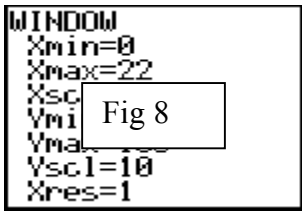


Fig 8

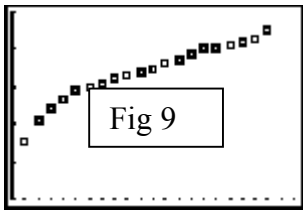


Fig 9

3. Predict how the graph of the modified scores would compare to the graph of the original scores. Graph the modified scores on the same graph as the original scores and check your predictions.
   Since 5 points were added to each score, the graph of the modified scores should be a vertical translation of 5 units of the graph of the original scores

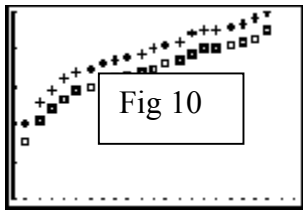


Fig 10

4. If y = f(x) represents the function rule for the original scores, determine a representation for the function rule for the modified scores. Explain your answer.
   The function rule of the modified scores is y = f(x) + 5 because the graph is shifted 5 units up

Extension Questions
Solutions

5. Explain algebraically why the mean shown for the modified list is 5 points higher that the mean for the original list.

The sum of the scores of the original list may be represented by $\sum_{i=1}^{20} s_i$ *where* s_i *is each individual score added. Because there are 20 scores the mean is represented by* $\frac{1}{20}\sum_{i=1}^{30} s_i$. *Where 5 is added to each score, the sum of the score is represented by* $\sum_{i=1}^{20}(s_i + 5)$. *The new mean is*

$$\sum_{i=1}^{20}(s_i + 5) = \frac{1}{20}\sum_{i=1}^{20}(s_i) + \frac{1}{20}\sum_{i=1}^{20}(5) = \frac{1}{20}\sum_{i=1}^{20}(s_i) + \frac{1}{20}20(5) = \frac{1}{20}\sum_{i=1}^{20}(s_i) + 5$$

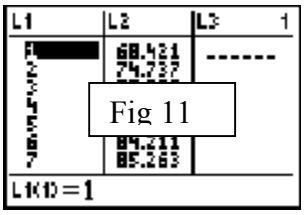
The new mean is the original mean plus 5.

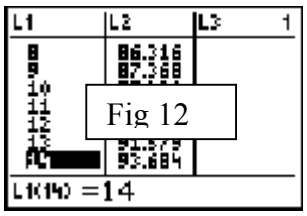
6. One of the judges was uncomfortable with the method of modifying the scores by adding 5 points to each score. Instead, he suggested that perhaps the scores should have been multiplied by a factor that would make the highest scores equal 100 points. The judges decided to try this method and compare the results.

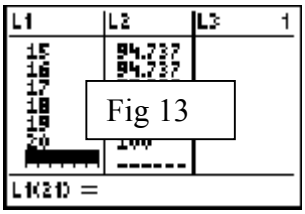
   What factor should be used to multiply the highest original score to obtain 100?

The highest original score was 95. Multiplying the number by $\frac{100}{95}$ *will produce a score of 100.*

7. Create a table showing the new scores. How does the mean of the original compare with that of the second modification of the scores? Explain any differences you find between the comparison of the original scores and the second modification.







The original scores in L2 were each multiplied by $\frac{100}{95}$ *to produce the newly modified scores.*

The mean of the second modification of the scores is found by adding all of the scores (1,762.105263), and dividing the sum by 20. The mean of the second modification is 88.1 (rounded to the nearest tenth).

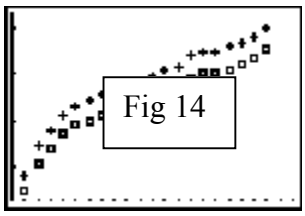
The scores on the second modification are "stretched" by a factor of $\frac{100}{95}$. All of the score increase, so the mean increases. However, the increase does not follow the same pattern as in the first modification. The first modification involves all scores being raised by the same value; the mean increases by the exact same value also. In the second modification, the scores are multiplied by a factor of $\frac{100}{95}$. This results in the new mean being $\frac{100}{95}$ times the original mean. This can be shown algebraically.

The sum of the scores of the original list may be represented by $\sum_{i=1}^{20} s_i$. The mean is represented by $\frac{1}{20}\sum_{i=1}^{30} s_i$. The mean of the scores multiplied by $\frac{100}{95}$ is

$\frac{1}{20}\sum_{i=1}^{30}\frac{100}{95}s_i = \frac{1}{20}\frac{100}{95}\sum_{i=1}^{30}s_i = \frac{100}{95}\left(\frac{1}{20}\sum_{i=1}^{30}s_i\right)$. *The new mean of the scores is the original mean multiplied by $\frac{100}{95}$.*

8. Graph the newly modified scores on the same graph with the original scores. Describe the change in the graph from the original to the newly modified scores.

The original scores are represented by squares. Dots represent the final modification produced by multiplying each score by $\frac{100}{95}$. The spread between the scores on the upper end of the graph is visually evident as being bigger than the spread on the lower end of the two graphs.



9. Which method do you think the judges should use the meet the competition guidelines of rating the highest artwork a 100? Justify your reasoning.

Answers will vary. For example, one might say that the method of adding 5 points is fairer because it helps each student by adding the same number of point to each score. Another student might say that the amount added should vary. More points should be added to the higher scores because they did better.

Teaching notes

This task can be assigned as homework. Participants are to bring their completed homework and compare findings in groups of four. Any discrepancies in the groups should be addressed with the whole class. Some questions to ask as the instructor monitors group conversations:

Q What do you have to do to create a table showing the new scores? Q
Q How can you find the mean for this set of data? Q
Q How can you determine median for this set of data? Q

See Algebra II Assessments from the Dana Center for sample student work.

Technology notes

An alternative method of finding the mean and median can be found using one variable statistics on the graphing calculator. The process for L1 is shown below. Follow the same procedure for L3.

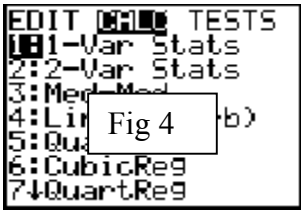


Fig 4

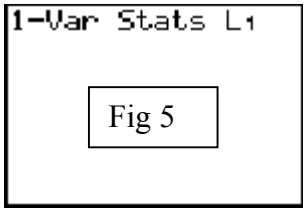


Fig 5

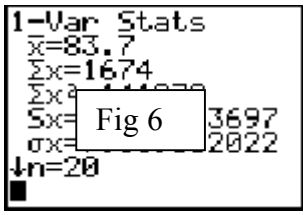


Fig 6

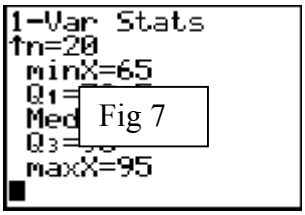


Fig 7

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