

Task 1.3.5: TREE GROWTH (PART I)**Solutions**

The table below shows the board feet in hundreds of a certain type of tree by the age of the tree.

Age of tree	100s of board feet per tree
0	0
20	1
40	6
80	33
100	56
120	88
160	182

Instruct participants to enter the data into their graphing calculators and create a scatter plot.

1. Make a scatter plot using L1 and L2. Without using the regression feature of the calculator find a cubic equation that models the data.

Refrain from using the regression to find an equation that models the trend. Encourage participants to simply guess and check an equation that models the trend by graphing $y = ax^3$ and then adjust the compression factor, a , until it is a reasonable equation that models the trend. This is not a “best fit model.” It is an equation that models a trend.

Fig 1

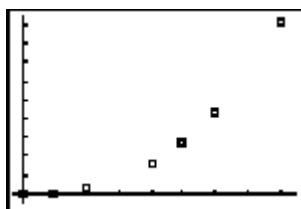


Fig 2

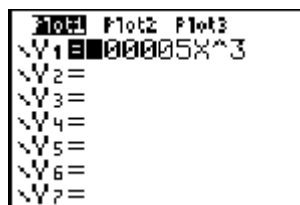
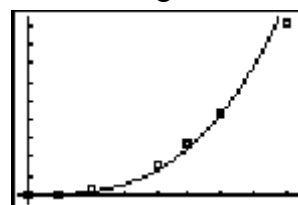


Fig 3



2. What is the stretch/compression coefficient?

.00005 If participants are having difficulty suggest that they start with $y = x^3$. Have them evaluate the parent function for $x = 20$. The y -value is 8000. It needs to be one, so how can 8000 be brought down to 1? $8000/8000 = 1$. They could try using $1/8000$ for the coefficient and adjust from there.

What is the geometric term for the compression coefficient?

Dilation.

What is the y -intercept?

Zero.

How do each relate to the problem?

.000005 relates to the growth rate. The tree does not have any height when the seed is planted.

3. Write the function as a transformation of the parent function

$$y = x^3. y = .00005x^3.$$

4. What is a reasonable domain for this situation? Is there an upper limit (How long do trees live)?

$x \geq 0$. Answers will vary.

Task 1.3.5: TREE GROWTH (PART II)**Solutions**

Suppose that a timber company plants young trees on cleared land. They want to know the board feet per tree in terms of the age of the forest. Assume that the trees are five years old when they are planted.

1. Make a prediction of how the new graph will look in relation to the original graph.

Answers will vary. Many participants will predict a vertical shift. Allow them to make their predictions without leading them. Discuss labeling the axes. The horizontal axis is the age of the forest and the vertical axis is 100s of board feet per tree.

2. Set up a new table of values with age of forest being the independent variable and board feet per tree in hundreds being the dependent variable. How old will the forest be before the harvesting can begin?

Fifteen years old. Ask this question: Do you see that information on the graph? Yes at (15, 1)

Age of forest	100s of board feet per tree
-5	0
15	1
35	6
75	33
95	56
115	88
155	182

3. Set up a scatter plot of the new data. Use L3 and L4. How can you use operations on L1 to find L3?

Fig 4

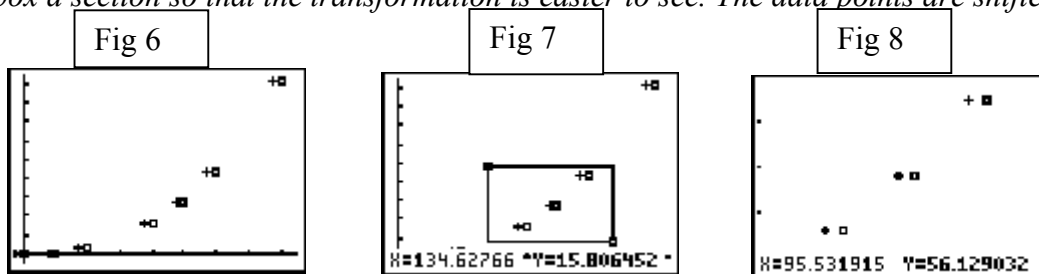
L2	L3	L4	3
0	0	0	
1	1	1	
6	6	6	
33	33	33	
56	56	56	
88	88	88	
182	182	182	

$L3 = L1 - 5$

Fig 5

Plot1	Plot2	Plot3
Off	Off	Off
Type: []	Type: []	Type: []
Xlist: L3	Xlist: L3	Xlist: L3
Vlist: L4	Vlist: L4	Vlist: L4
Mark: []	Mark: []	Mark: []

4. View both sets of data on one screen. Describe the transformation that occurs. Zoom box a section so that the transformation is easier to see. The data points are shifted to the left.



5. Write a function that models that data. $y = .00005(x + 5)^3$
 What transformation do you need from the original function?

$x + 5$. This is a horizontal shift to the left.

Is the resulting transformation the one that you predicted when you drew the prediction graph? If not, explain the difference?

Answers will vary. Since we subtracted five from the independent variable, we have a horizontal shift. We shifted the time from which we calculated the board feet.

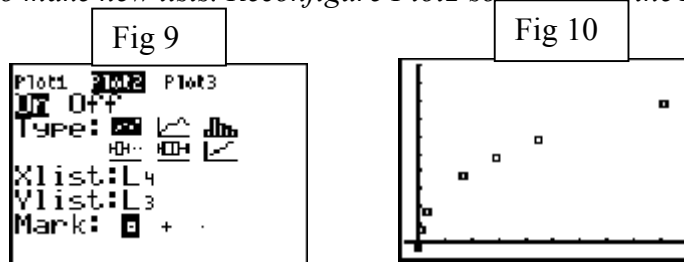
6. What is the meaning of the age of the forest when board feet per tree in hundreds are zero?
 $(-5, 0)$ The forest is in some other state at that time. The tree seedlings are in a nursery. This is the time five years before the forest is planted with young trees.

Task 1.3.5: TREE GROWTH (PART III)**Solutions**

Suppose you use hundreds of board feet per tree as the independent variable in the forest problem.

- Using the data in L3 and L4 set up a scatter plot so that hundreds of board feet per tree is the independent variable.

It is not necessary to make new lists. Reconfigure Plot2 so that L4 is the Xlist and L3 is the Ylist.

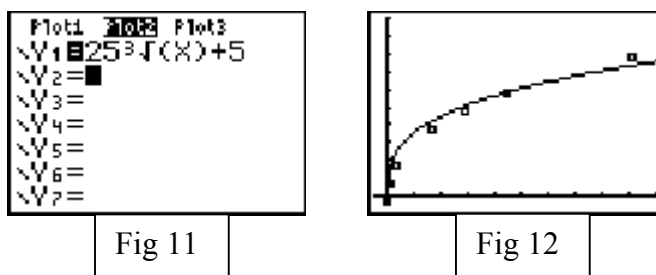


- Write an equation with hundreds of board feet per tree as the independent variable.
Participants may go through the same guess and check method that was used in Tasks 1.3.5 Parts I and II. Some may just adjust the numbers from the function found in Task 1.3.5. Part II.

$$y = 25\sqrt[3]{x} + 5.$$

Ask this question: What kind of function are we dealing with here?
The cube root function.

- How does this equation relate to the equation in Task 1.3.5 Part II?
It is the inverse function.



- What information do you note when the equation is written with hundreds of board feet per tree as the independent variable?

If you know the amount of board feet needed, you can find how old the tree has to be.

Math notes

In these tasks participants make scatter plots of time and board feet data from forestry information. Participants make conjectures, fit curves to the data, interpret results, and find inverses.

Teaching notes

Prior knowledge of transformations, independent and dependent variables, and inverse functions is required. The previous knowledge may be limited to linear and quadratic functions.

The big idea here is that we can use real data to see an application of the horizontal shift. We also review the cubic parent graph, the stretch/compression coefficient, estimating trends and the concept of independence and dependence. It is an introduction to inverse functions.

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1. Make a scatter plot using L1 and L2. Without using the regression feature of the calculator find a cubic equation that models the data.
2. What is the stretch/compression coefficient? What is the geometric term for the compression coefficient? What is the y-intercept? How do each relate to the problem?
3. Write the function as a transformation of the parent function $y = x^3$.
4. What is a reasonable domain for this situation? Is there an upper limit (How long do trees live)?

Task 1.3.5: TREE GROWTH (PART II)

Suppose that a timber company plants young trees on cleared land. They want to know the board feet per tree in terms of the age of the forest. Assume that the trees are five years old when they are planted.

1. Make a prediction of how the new graph will look in relation to the original graph.



2. Set up a new table of values with age of forest being the independent variable and board feet per tree in hundreds being the dependent variable. How old will the forest be before the harvesting can begin?

Age of forest	100s of board feet per tree

3. Set up a scatter plot of the new data. Use L3 and L4. How can you use operations on L1 to find L3?
4. View both sets of data on one screen. Describe the transformation that occurs.

5. Write a function that models that data. What transformation do you need from the original function? Is the resulting transformation the one that you predicted when you drew the prediction graph? If not, explain the difference?

6. What is the meaning of the age of the forest when board feet per tree in hundreds are zero?

Task 1.3.5: TREE GROWTH (PART III)

Suppose you use hundreds of board feet per tree as the independent variable in the forest problem.

1. Using the data in L3 and L4 set up a scatter plot so that hundreds of board feet per tree is the independent variable.
2. Write a function with hundreds of board feet per tree as the independent variable.
3. How does this function relate to the equation in Task 1.3.5 Part II?
4. What information do you note when the equation is written with hundreds of board feet per tree as the independent variable?