

## STRAND 1: FOUNDATIONS OF FUNCTIONS

### TOPIC 1.2: TRANSFORMATIONS

#### Topic Notes

#### Mathematical focus

Participants will develop intuition and conceptual knowledge of functions via a transformation approach to graphing functions in terms of parent functions.

#### Topic overview

In this topic, participants will investigate the effects of various transformations of parent functions both quantitatively and qualitatively using graphing calculators, table building, and sketching. They will be asked to generalize their findings.

This topic includes 5 tasks:

- Task 1.2.1: Exploring Transformations
- Task 1.2.2: Domains
- Task 1.2.3: Transformations with Technology
- Task 1.2.4: Summarize Findings
- Task 1.2.5: Reflect and Apply

#### TE<sub>x</sub>ES standards focus

**TE<sub>x</sub>ES Standard II.004 Patterns and algebra.** The teacher uses patterns to model and solve problems and formulate conjectures. The beginning teacher:

- (A) Recognizes and extends patterns and relationships in data presented in tables, sequences, or graphs.

**TE<sub>x</sub>ES Standard II.005 Patterns and algebra.** The teacher understands attributes of functions, relations, and their graphs. The beginning teacher:

- (C) Understands that a function represents a dependence of one quantity on another and can be represented in a variety of ways (e.g., concrete models, tables, graphs, diagrams, verbal descriptions, symbols).
- (E) Applies basic transformations [e.g.,  $k f(x)$ ,  $f(x) + k$ ,  $f(x - k)$ ,  $f(kx)$ ,  $|f(x)|$ ] to a parent function,  $f$ , and describes the effects on the graph of  $y = f(x)$ .

**TE<sub>x</sub>ES Standard V.019 Mathematical processes and perspectives.** The teacher understands mathematical connections both within and outside of mathematics and how to communicate mathematical ideas and concepts.

- (C) Translates mathematical ideas between verbal and symbolic forms.
- (D) Communicates mathematical ideas using a variety of representations (e.g., numeric, verbal, graphical, pictorial, symbolic, concrete).

**TEKS/TAKS focus**

**TEKS 2A.4 Algebra and geometry.** The student connects algebraic and geometric representations of function. The student is expected to :

(A) identify and sketch graphs of parent functions, including linear ( $f(x) = x$ ), quadratic ( $f(x) = x^2$ ), exponential ( $f(x) = a^x$ ), and logarithmic ( $f(x) = \log_a x$ ) functions, absolute value of  $x$  ( $f(x) = |x|$ ), square root of  $x$  ( $f(x) = \sqrt{x}$ ), and reciprocal of  $x$  ( $f(x) = 1/x$ ); and

(B) extend parent function with parameters such as  $a$  in  $f(x) = a/x$  and describe parameter changes on the graph of parent functions.

**TEKS 2A.7 Quadratic and square root functions.** The student interprets and describes the effects of changes in the parameters of quadratic functions in applied and mathematical situations. The student is expected to:

(B) use the parent function to investigate, describe, and predict the effects of changes in  $a$ ,  $h$ , and  $k$  on the graphs of  $y = a(x - h)^2 + k$  form of a function in applied and purely mathematical situations.

**High School TAKS Objective 1:** The student will describe functional relationships in a variety of ways.

**High School TAKS Objective 2:** The student will demonstrate an understanding of the properties and attributes of functions.

**Materials**

	Task 1	Task 2	Task 3	Task 4	Task 5
<b>Graphing calculator</b>	*				*
<b>Transparencies</b>	*	*	*		
<b>Transparency pens</b>	*	*	*		
<b>Markers</b>				*	
<b>Wall chart paper</b>				*	

**Procedure**

In Algebra II, the foundation for developing a connected view of function building is begun in this series of tasks. Previously, we often taught each line and each parabola in Algebra II as separate objects with different properties (slopes, vertices, etc.). Instead, students need to develop a conceptual understanding of certain types of polynomials and their properties and then move to a view of functions as transformations of a an initial (or parent) function (such as  $f(x) = x$  for linear functions or  $f(x) = x^2$  for quadratic functions).

When facilitating this topic, consider the following suggestions. Begin by having the participants work in small groups on Task 1 and then discuss their findings as a whole group. Follow the same procedure for Task 2. Task 3 is lengthy, but it is important to give participants the opportunity to generalize their findings and compare their findings. You may have participants work in groups such that half of the class does parts a and b of

each question and then the other half does parts c and d. Then groups compare findings and make generalizations. Participants should work together on Task 4.

### Summary

This topic focuses on a global view of the effects of transformations on the graph of an initial (or parent) function instead of on algebraic manipulation of functions. This central idea is carried throughout the tasks. Development of this ability to sketch a polynomial function based upon a few initial conditions and knowing its degree is an important foundation for curve sketching in calculus and precalculus.

### Extensions

This topic could be extended in many ways; however, in the context of this module (course) we suggest that any extensions be revisited if time permits at the end of the module. A good extension would be to ask participants to consider whether or not they could sketch or design functions such that  $f(x) = f(x + 2)$ ,  $f(x) = f(x - 2)$ ,  $f(x) = 2f(x)$ ,  $f(x) = 1/2f(x)$ ,  $f(x) = f(2x)$ ,  $f(x) = f(1/2x)$ ,  $f(x) = f(x) + 2$ ,  $f(x) = f(x) - 2$ ,  $f(x) = f(-x)$ ,  $f(x) = |f(x)|$ ,  $f(x) = -f(x)$ ,  $f(x) = f(|x|)$ .

### Assessments

Task 5 would be an appropriate assessment which can be given as a homework assignment. Also, a group assessment on the following day (to be completed on wall chart paper by each group) would be to have participants sketch, make a table, and describe in words the transformation of  $f$  when  $y = 2f(3(x - 1)) + 1$ . Then, have them summarize the effects of  $y = af(b(x - c)) + d$  on the parent function  $f$ .

Transitions to the classroom—which of the tasks are appropriate only for teachers? Which for students? How would we modify a teacher-task to make it student appropriate?

	Teacher use only	Modify for students	Ready for students
Task 1			*
Task 2			*
Task 3		*	
Task 4		*	
Task 5		*	