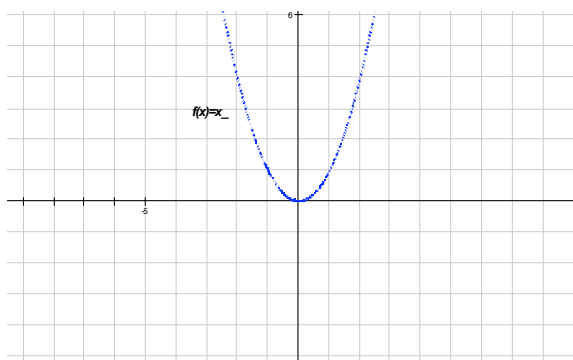
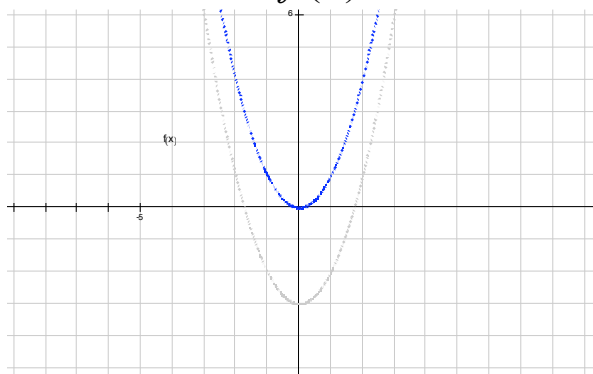


TASK 1.1.3: EXPLORING NOTATION DIFFERENCES IN GEOMETRY AND ALGEBRA**Solutions**

1.

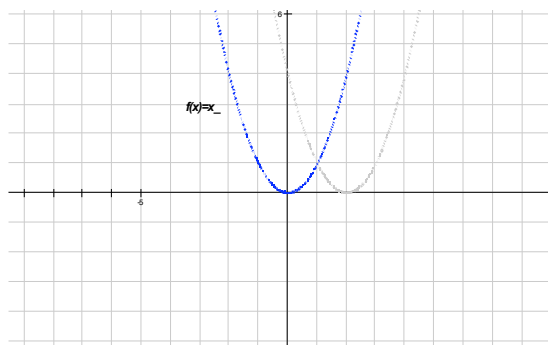


a) Sketch the function , $f(x) = x^2$, translated down 3 units.



- b) How would this translation be written in coordinate notation? $T : (x, y) \rightarrow (x, y - 3)$
- c) How do you think this translation would be written using function notation?
 $g(x) = x^2 - 3$ or $g(x) = f(x) - 3$
- d) Check your conclusion by graphing the original function and the transformed function and verify that it matches your graph above. Did it match? Why or why not? **The graph on the calculator should match the sketch on part a. The translation is written by subtracting 3 from the original function. Both shift the y-values down by 3.**

2.



- a) Describe the transformation of $f(x) = x^2$ to its image (lighter graph).

f is translated right 2.

- b) How would this transformation be written in coordinate notation?

$$T : (x, y) \rightarrow (x + 2, y)$$

- c) How do you think this transformation would be written using function notation?

Possible answers: $g(x) = (x - 2)^2$ or $g(x) = f(x - 2)$, also some participants will probably propose the **incorrect** answer $g(x) = (x + 2)^2$ or $g(x) = f(x + 2)$

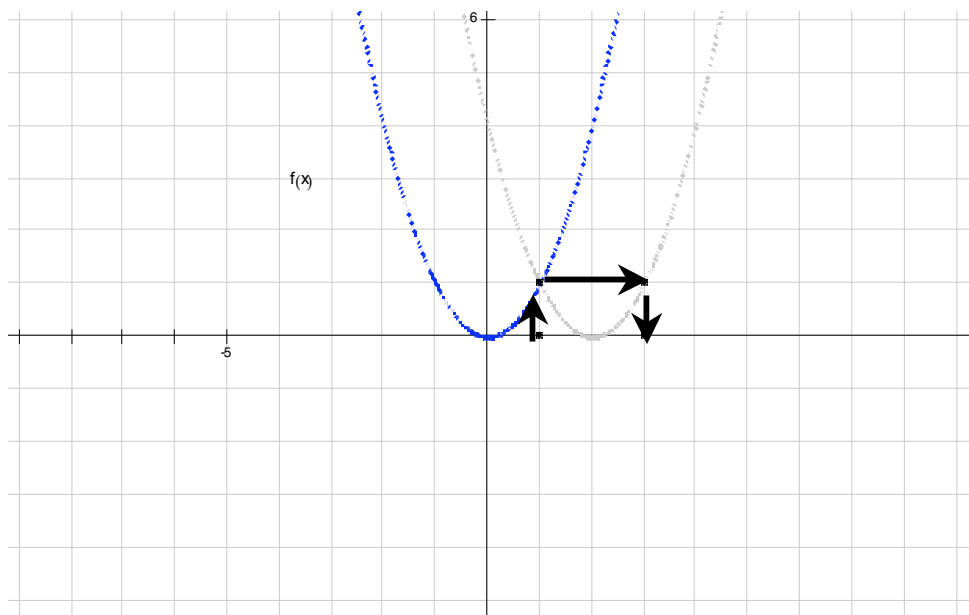
- d) Check your conclusion by graphing the original function and the transformed function and verify that it matches your graph above. Did it match? Why or why not?

Depending upon their answers to part c, participants will either see that their graphs matched or that they didn't. If they didn't match they'll have recognized that they used "x+2" in the function notation rather than "x-2". If they do match, the explanation may center about the following line of reasoning: One way to explain this is by examining the meaning of the coordinate notation. When we write the coordinate notation with the function value $f(x)$ for y we note that: $T : (x, f(x)) \rightarrow (x', f(x))$ where x' refers to the transformed image of x . This is not the usual way in which we would denote points on the graph of a function. That is, the second coordinate is usually written in terms of an expression using the first coordinate. Thus, participants may be led to think about how they could write x in terms of x' .

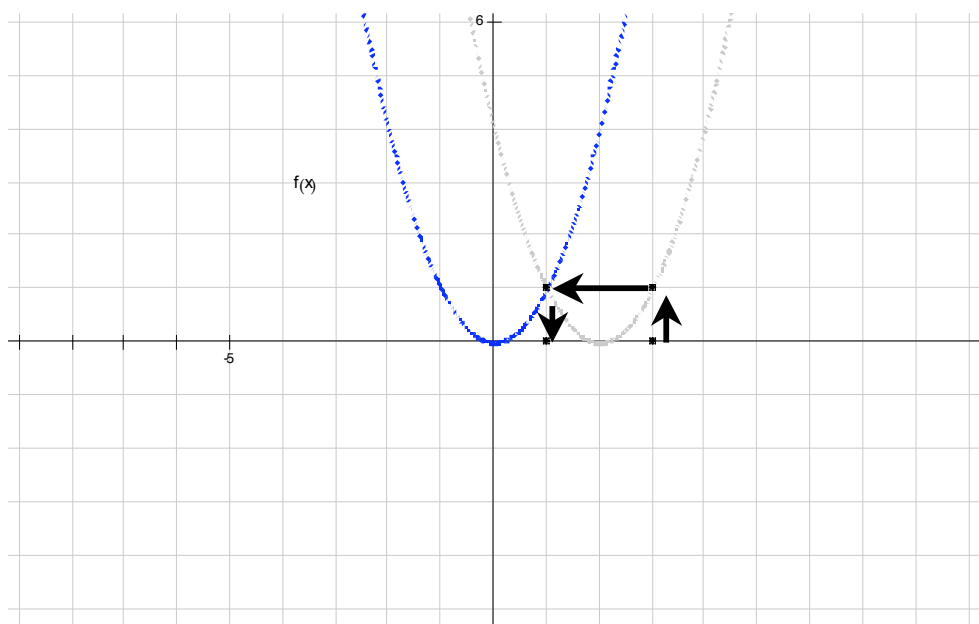
Another way to explain the situation is that the notation is dependent on your point of reference.

In geometry, the original graph is your beginning point of reference. The translation is written so that its coordinates refer to how the original graph was transformed or changed to produce the new. See the graph below for pictorial representation of this:

Algebra II: Strand 1. Foundations of Functions; Topic 1. Linking Foundations; Task 1.1.3



Using function notation, the point of reference is different. You begin at the transformed image's domain and range value and reference back to the original location or the preimage. (As shown below in the figure below.)



Analyzing and comparing this notation with b's answer, you could say that

$$x' = x + 2$$

$$x' - 2 = x$$

So that the same domain value used the notation is consistent we can make the following substitution:

$$T : (x, f(x)) \rightarrow (x', f(x' - 2))$$

Looking at the notation in this way then it makes sense in function notation that the translation right is written as $g(x) = (x - 2)^2$ or $g(x) = f(x - 2)$

Math notes

The questions in Task 1.1.3 were written to set up a conversation about the differences in notation between geometry and algebra when making horizontal transformations, and to make a transition into the tasks for the Foundation of Function Strand. The apparent difference in treatment of horizontal transformations in geometry and algebra is almost never discussed by most texts and most practicing teacher have not had the opportunity to investigate this difference.

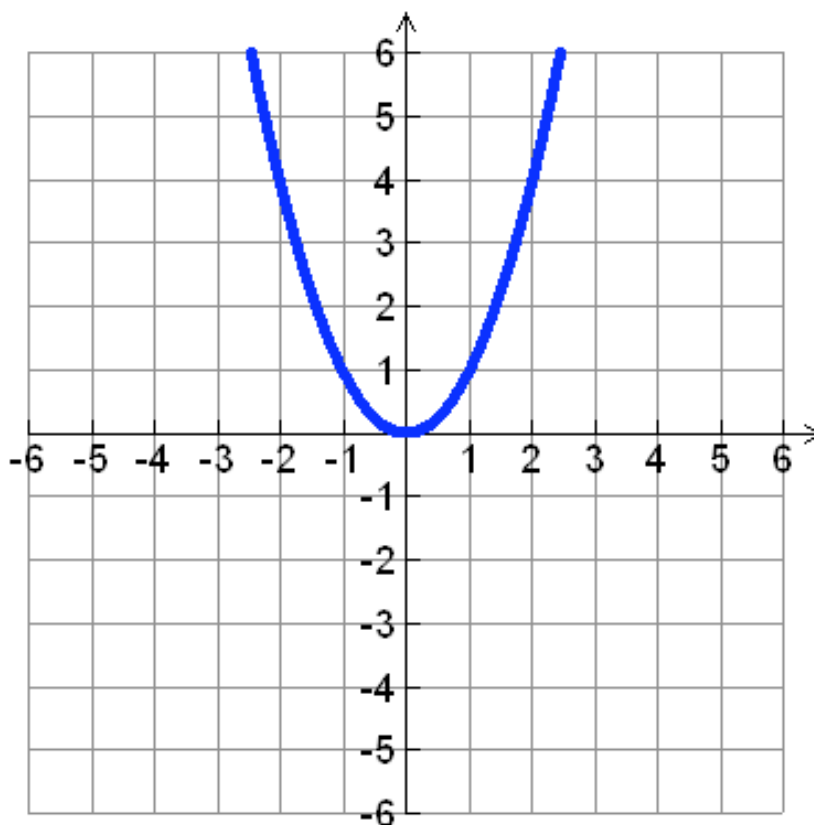
Teaching notes

For this task, have participants work in groups of three to four. Exercise 1 is relatively straightforward for participants and they will not see any significant difference in how the vertical translation is represented using coordinate notation and function notation.

In Exercise 2, participants will (re-)discover that horizontal transformations represented in coordinate notation may cause confusion when using function notation. Participants need to be given a little time to try to come up with some type of plausible explanation for the differences in notation. **Suggestion:** After participants have had a chance to discuss Exercise 2 in their groups, assign for *homework* “**There seems to be a contradiction between function notation for horizontal shifts and coordinate notation. Explain why this is not a contradiction.**” The following meeting, have participants share their individual responses (have them pass their papers clockwise in their groups until each has read the papers of all the members of their group). Pass out chart paper and have each group construct a “group answer” to the homework question. Have them post their answers and then discuss them with the whole class. The results from this are very informative for the instructor and they underscore why students have difficulty with transformations and function notation. Instructors may find that many of the participants will have never thought about how to explain this to their students.

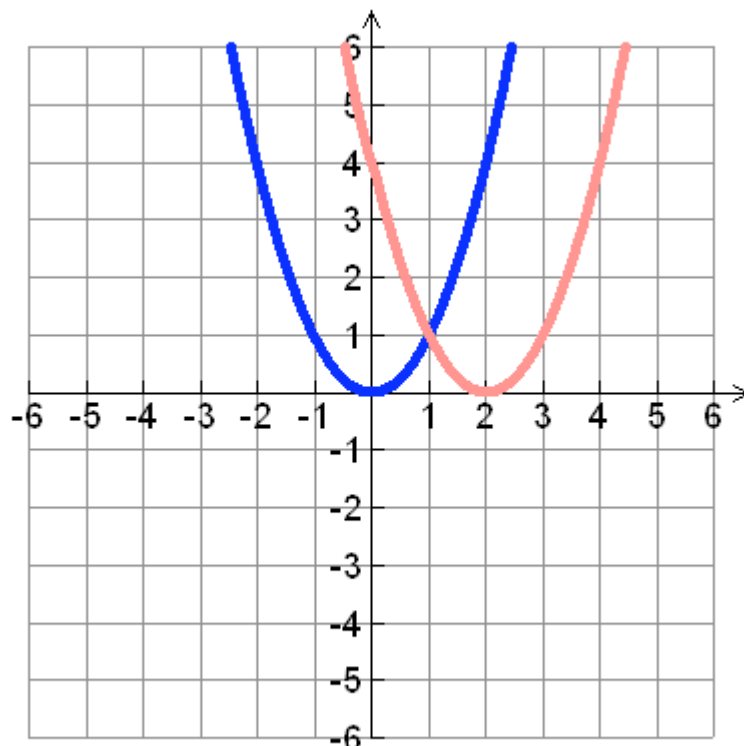
TASK 1.1.3: EXPLORING NOTATION DIFFERENCES IN GEOMETRY AND ALGEBRA

1.



- Sketch the function, $f(x) = x^2$, translated down 3 units.
- How would this translation be written in coordinate notation?
- How do you think this translation would be written using function notation?
- Check your conclusion by graphing the original function and the transformed function and verify that it matches your graph above. Did it match? Why or why not?

2.



- Describe the transformation of $f(x) = x^2$ to its image (lighter graph).
- How would this transformation be written in coordinate notation?
- How do you think this transformation would be written using function notation?
- Check your conclusion by graphing the original function and the transformed function and verify that it matches your graph above. Did it match? Why or why not?