

STRAND 2: LINEAR FUNCTIONS**TOPIC 2.4: STACK OF CUPS****Topic Notes****Mathematical focus**

Further development of linear functions includes investigation of the function $y = mx + b$. Using contextual situations, students investigate the effect of the addition of a non-zero constant to the dependent variable, x .

Terms: function, independent variable, dependent variable, domain, range, h-intercept, rate of change, constant rate, proportional, direct variation, discrete, continuous

Topic overview

This topic contains 8 tasks:

Task 2.4.1: The Height Function

Task 2.4.2: The Range and Relationship of the Size Parameters C and S

Task 2.4.3: Boxes of Cups: Solving Equations Based on the Height Function

Task 2.4.4: The Count Function

Task 2.4.5: The Relationship of the Height Function and the Count Function

Task 2.4.6: “Stack Space” and “Cup Space”

Task 2.4.7: Direct Proportionality

Task 2.4.8: Paths Between Graphs

In these exercises, participants complete the activity with minimal instruction. After they complete each exercise, participants share and discuss the exercise in depth, especially the mathematics and the scaffolding questions required to guide students to an in-depth understanding. Participants collect, organize, and graph data. They investigate the effect of addition of a constant to the variable term on the graph of the function, $y = mx$. As they investigate the function $y = mx + b$, they further develop their understanding of independent variable, dependent variable, domain, range, proportional, and direct variation. They translate from and among concrete models, tables, graphs, verbal descriptions, and algebraic rules.

In addition, participants consider a solution to a specific case of a functional relationship, the inverse of a function, relationship of a function and its inverse, and the meaning of the paths between a graph of a function and its inverse.

Finally, in this activity participants consider different interpretations of three parameters and possible relationships among the parameters.

This activity includes the interaction of seven different algebraic concepts through an investigation of one concrete context.

TEExES standards focus

TEExES Standard II.004 Patterns and algebra. The teacher uses patterns to model and solve problems and formulate conjectures. The beginning teacher:

- (A) Recognizes and extends patterns and relationships in data presented in tables, sequences, or graphs.

TEExES Standard II.005 Patterns and algebra. The teacher understands attributes of functions, relations, and their graphs. The beginning teacher:

- (A) Understands when a relation is a function.
- (B) Identifies the mathematical domain and range of functions and relations and determines reasonable domains for given situations.
- (C) Understands that a function represents a dependence of one quantity on another and can be represented in a variety of ways (e.g., concrete models, tables, graphs, diagrams, verbal descriptions, symbols).

TEExES Standard II.006 Patterns and algebra. The teacher understands linear and quadratic functions, analyzes their algebraic and graphical properties, and uses them to model and solve problems. The beginning teacher:

- (A) Understands the concept of slope as a rate of change and interprets the meaning of slope and intercept in a variety of situations.
- (F) Solves problems involving quadratic functions using a variety of methods (e.g., factoring, completing the square, using the quadratic formula, using a graphing calculator).
- (G) Models and solves problems involving linear and quadratic equations and inequalities using a variety of methods, including technology.

TEKS/TAKS focus

TEKS A.1 Foundations for functions. The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways. The student is expected to:

- (A) describe independent and dependent quantities in functional relationships;
- (B) gather and record data and use data sets to determine functional relationships between quantities;
- (C) describe functional relationships for given problem situations and write equations or inequalities to answer questions arising from the situations; and
- (D) represent relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities.

High School TAKS Objective 1: The student will describe functional relationships in a variety of ways.

TEKS A.2 Foundations of functions. The student uses the properties and attributes of functions. The student is expected to:

- (B) identify mathematical domains and ranges and determine reasonable domain and range values for given situations, both continuous and discrete.

High School TAKS Objective 2: The student will demonstrate an understanding of the properties and attributes of functions.

TEKS A.5 Linear functions. The students understand that linear functions can be represented in different ways and translates among their various representations. The student is expected to:

- (A) determine whether or not given situations can be represented by linear functions; and
- (C) use, translate, and make connections among algebraic, tabular, graphical, or verbal descriptions of linear functions.

TEKS A.6 Linear functions. The student understands the meaning of the slope and intercepts of the graphs of linear functions and zeros of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations. The student is expected to:

- (A) develop the concept of slope as rate of change and determine slopes from graphs, tables, and algebraic representations;
- (B) interpret the meaning of slope and intercepts in situations using data, symbolic representations, or graphs;
- (D) graph and write equations of lines given characteristics such as two points, a point and a slope, or a slope and y-intercept; and
- (E) determine the intercepts of the graphs of linear functions and zeros of linear functions from graphs, tables, and algebraic representations.

High School TAKS Objective 3: The student will demonstrate an understanding of linear functions.

Materials

Task	2.4.1	2.4.2	2.4.3	2.4.4	2.4.5	2.4.6	2.4.7	2.4.8
Cups	x	x	x	x	x	x	x	x
Stack of Cups Handout	x	x	x	x	x	x	x	x
Rulers	x	x	x	x	x	x	x	x
Graph paper	x	x	x	x	x	x	x	x
Graphing calculators	x	x	x	x	x	x	x	x

Procedure

1. Arrange participants in groups of 3 to 4. Participants need a variety of cups—styrofoam cups—6 ounces, 12 ounces, 16 ounces, plastic cold beverage, plastic glasses, etc. Distribute one copy of *Stack of Cups* handout to each participant.

Participants investigate the relationship between the number of cups in a stack and the height of the stacks. Ask participants to complete in groups Task 2.4.1: The Height Function. This activity moves participants intuitively from a verbal description of a situation to a rule and a graph representing the situation. In addition, participants investigate domain, range, independent variable, dependent variable, discrete, continuous, rate of change, constant rate, h-intercept, and slope.

After the participants have completed this activity, they should share and discuss their findings.

2. Provide time for participants to complete Task 2.4.2: The Range and Relationship of the Size Parameters C (cup height) and S (stick-up height). In this activity participants compare the different possible sizes for C and S.

After the participants have completed this activity, they should share and discuss their findings.

3. Provide time for participants to complete Task 2.4.3: Solving Equations Based on the Height Function. In this activity participants write and solve an equation and interpret the solution for a situation that limits the height of the stack of cups.

After the participants have completed this activity, they should share and discuss their findings.

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4. Provide time for participants to complete Task 2.4.4: The Count Function. In this activity participants investigate the inverse of $h = (1.5 \text{ cm})n + 7.5 \text{ cm}$; that is, for a given height, h , of the stack, what is the number, n , of cups.

After the participants have completed this activity, they should share and discuss their findings.

5. Provide time for participants to complete Task 2.4.5: The Relationship of the Height Function and the Count Function. In this activity participants investigate the relationship of the height function and the count function from graphs of each on the same coordinate system.

After the participants have completed this activity, they should share and discuss their findings.

6. Provide time for participants to complete Task 2.4.6: “Stack Space” and “Cup Space”. In this activity students investigate the difference in the role of the variables n and h (called **stack space**), and the parameters B and S (called **cup space**).

After the participants have completed this exercise, they should share and discuss their findings.

7. For Task 2.4.7, provide time for participants to complete Task 2.4.7: Paths Between Graphs. In this activity participants consider two different types of cups from a graphical representation. From the graphical representation, they write verbal descriptions of various components of the graph. For example, what is the interpretation of the horizontal distance on the graph for the same ordinate and the vertical distance on the graph for the same abscissa?

Summary

Through the tasks in this topic, participants will become acquainted with a situation that is modeled by the standard linear model $y=mx+b$. The topic emphasizes the connection among concrete, verbal, symbolic, tabular, and graphic representations of the linear model.

Extensions

This task is paralleled by a task included in *TEXTEAMS Algebra I: 2000 and Beyond* called the *Valentine’s Day Problem*. The *TEXTEAMS Algebra I Assessments* provide numerous problem solving situations for linear data that can be used as open ended assessments or extensions for this topic.

Assessment/Transition to the Classroom

Participants complete the Teacher’s Journal, record their responses, and make any modifications necessary to the tasks for them to be completely classroom ready.