

TASK 2.4.6: “STACK SPACE” AND “CUP SPACE”

Solutions

So far we have the **variables** n and h and the **parameters** height of the cup without the stick-up, B , and the stick-up height, S . In this section, we will emphasize the difference in these roles by seeing what happens when they are reversed. For a reason that will become clear, we will refer to the n - h coordinate system in which we drew our graph in #4 as “stack space”.

Part I:

The cups we have been considering have values $B = 7.5$ cm and $S = 1.5$ cm. We know that a stack with a count of 61 has height of 99 cm. Call this a “(61, 99) stack”.

1. Show that cups of another size could also form a (61, 99) stack. (That is, show that cups with different values of the parameters B and S could still form a (61, 99) stack.)

Stacks of other types of cups can also have count $n = 61$ and height $h = 99$ cm. This can be seen from the formula $h = Sn + B$. We know that $61(1.4) + 13.6 = 99$. $61S + B = 99$ will hold for other values of B and S by making the S parameter larger and the B parameter smaller or making the S parameter smaller and the B parameter larger. Specific equations can be found in 6c.

2. Choose the graphs of two different functions that include the point (61, 99) and explain difference in slope and h -intercept as they relate to this problem.
Various functions can be selected.

Two possible equations are: $h = 1.2n + 25.8$ and $h = 0.2n + 86.8$

Interpretation:

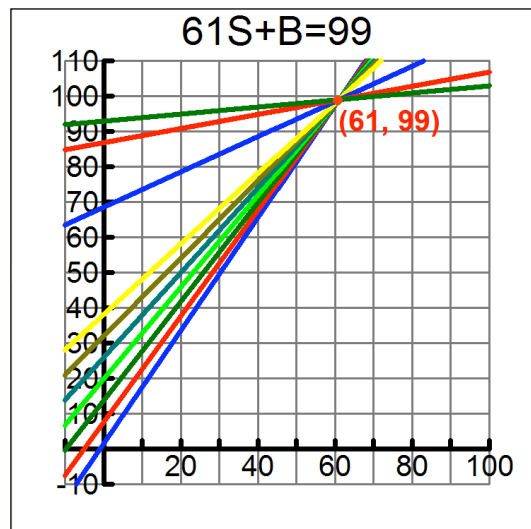
The slope (S) of the functions represents the measure of the stick-up height of the cups and the h -intercept (B) is the measure of the height of the base of the cups (that is, without the stick-up). The difference in B values (the h -intercepts of the two functions) is the difference in the heights of the bases of the two types of cups. The difference in S values (the slopes of the functions) is the difference in the measure of the stick-up heights of the two types of cups.

Algebra I: Strand 2. Linear Functions; Topic 4. Stack of Cups; Task 2.4.6

3. Using the coordinate grid draw the **height function** graphs of some of these other cup types that have a (61, 99) stack. Include the extreme cases: the graph with greatest possible slope and a graph with a very small slope. All the graphs should include the point (61, 99).

Below are graphs of some different types of cups that all include $n = 61$ and $h = 99$ (that is, 61 cups will measure 99 cm tall). All scatterplots of the situation are discrete graphs, however, the mathematical function is represented by a line graph.

$$\begin{aligned} h &= 1.6n + 1.4 \\ h &= 1.5n + 7.5 \\ h &= 1.4n + 13.6 \\ h &= 1.3n + 19.7 \\ h &= 1.2n + 25.8 \\ h &= 1.1n + 31.9 \\ h &= 1.0n + 38 \\ h &= 0.5n + 68.5 \\ h &= 0.2n + 86.8 \\ h &= 0.1n + 92.9 \end{aligned}$$



4. Show that in the n - h coordinate system, which we are calling **stack space**:

- a line graph represents a type of cup (a particular S and B).
- a point represents a type of stack (a particular n and h).

(The latter fact is why we called the n - h coordinate system **stack space**.)

In the n - h coordinate system, the line graph represents a specific type of cup that has a specified B (height of the base of the cup) and S (stick-up height of the cup). S is the slope of the line and B is the h -intercept of the line.

In the n - h system, a point on a line represents a particular stack, that is, the count and height.

Part II:

5. In the questions in Part I, the height of the base of the cup (B) and the stick-up height of the cup (S) were changed while the number of the cups (n) and the total height (h) of the stack of cups were kept constant (61, 99). In part II, the height of the base of the cup (B) and the stick-up height of the cup (S) are held constant while the parameters n (the number of cups) and h (the height of the stack of cups) change.

a. Starting with the function rule $h = 1.5n + 7.5$, find a new function rule for B in terms of S . Explain each step of your transformation in context.

Start with the function

$$h = 1.5n + 7.5 \quad (\text{height of the stack is } 1.5 \text{ cm per cup plus the base height, } 7.5 \text{ cm})$$

$$99 = 1.5(61) + 7.5 \quad (\text{that is, for space stack } (61, 99), \text{ a stack of cups } 99 \text{ cm tall is } 1.5 \text{ cm stick-up height per cup for } 61 \text{ cups plus the base height of } 7.5 \text{ cm})$$

$$99 = 61S + B \quad (\text{a stack of cups } 99 \text{ cm tall consists of } 61 \text{ cups with } S \text{ stick-up height plus a base } B \text{ cm})$$

$$B = 99 - 61S \quad (\text{the base of the cups is } 99 \text{ cm minus } 61 \text{ stick-up heights})$$

In three steps the roles of the variables and the parameters are interchanged. This new function $B = 99 - 61S$ has as variables the dimensions S and B of a cup, and as parameters the count $n = 61$ cups and $h = 99$ cm total height.

b. What is the independent variable?

The independent variable is S (stick-up height of the cup).

c. What is the dependent variable?

The dependent variable is B (height of the base of the cup)

d. What is the domain of the problem situation?

The domain is $\left\{d \in \mathbb{R} \mid 0 \leq d \leq \frac{99}{61}\right\}$. Note $(99/61, 0)$ has meaning in the problem situation if cups are perfect cylinders with no part of the cup hidden when you stack them.

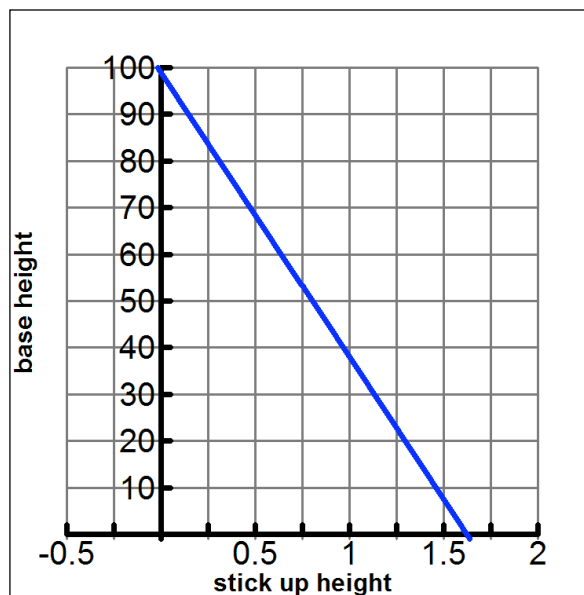
e. What is the range of the problem situation?

The range is $\left\{r \in \mathbb{R} \mid 0 \leq r < 99\right\}$. Note $(0, 99)$ does not have meaning in the problem situation; physically it is not possible to have cups that stack without a stick-up height if the cups are all alike.

Algebra I: Strand 2. Linear Functions; Topic 4. Stack of Cups; Task 2.4.6

6. a. On this new coordinate grid graph all the types of cup stacks such that 61 cups have a height of 99 cm.

*Below is a graph of the line $B = 99 - 61S$ in the S - B coordinates (**cup space**).*



- b. Is the graph of the problem situation continuous or not? Explain your thinking.

The graph is continuous; for any stick-up height there is a corresponding base height for 61 cups that form a stack 99 cm in height.

- c. What are the x- and y-intercepts of the line? What are the meanings of the intercepts in the problem situation?

For 61 cups that measure 99 cm tall, the x-intercept is the stick-up height in cm when the base height is 0; this is possible if cups are perfect cylinders with no part of the cup hidden when you stack them. The y-intercept is the base height when there is no stick-up height; physically this is not possible if the cups are all alike.

- d. Is this relationship a direct variation or indirect variation? Explain.

This relationship is not proportional nor is it a direct variation. The relationship has a constant rate of change, but does not include the point (0, 0).

7. Give a reason for using the title “cup space” for the new graph, and explain in general what:

- the line represents in cup space,
- a point represents in cup space.

*The line for **cup space** represents all the stacks of cups that contain count 61 and height of 99 cm.*

A point in cup space represents a particular type of cup, for example the cup type with $S = 1.5$ cm and $B = 7.5$ cm.

Algebra I: Strand 2. Linear Functions; Topic 4. Stack of Cups; Task 2.4.6

Given a coordinate $(0.6, 62.4)$ means if the stick-up is 0.6 cm and the base of the cup is 62.4 cm then it takes 61 cups to have a height of 99 cm.

Math notes

As an extension, ask participants to investigate the meaning of slope in **cup space** $(61, 99)$.

TASK 2.4.6: “STACK SPACE” AND “CUP SPACE”

In the stacks of cups problem, we are interested in the relationship between two variables h and n :

h = the height of the stack in cm

n = the number of cups in the stack

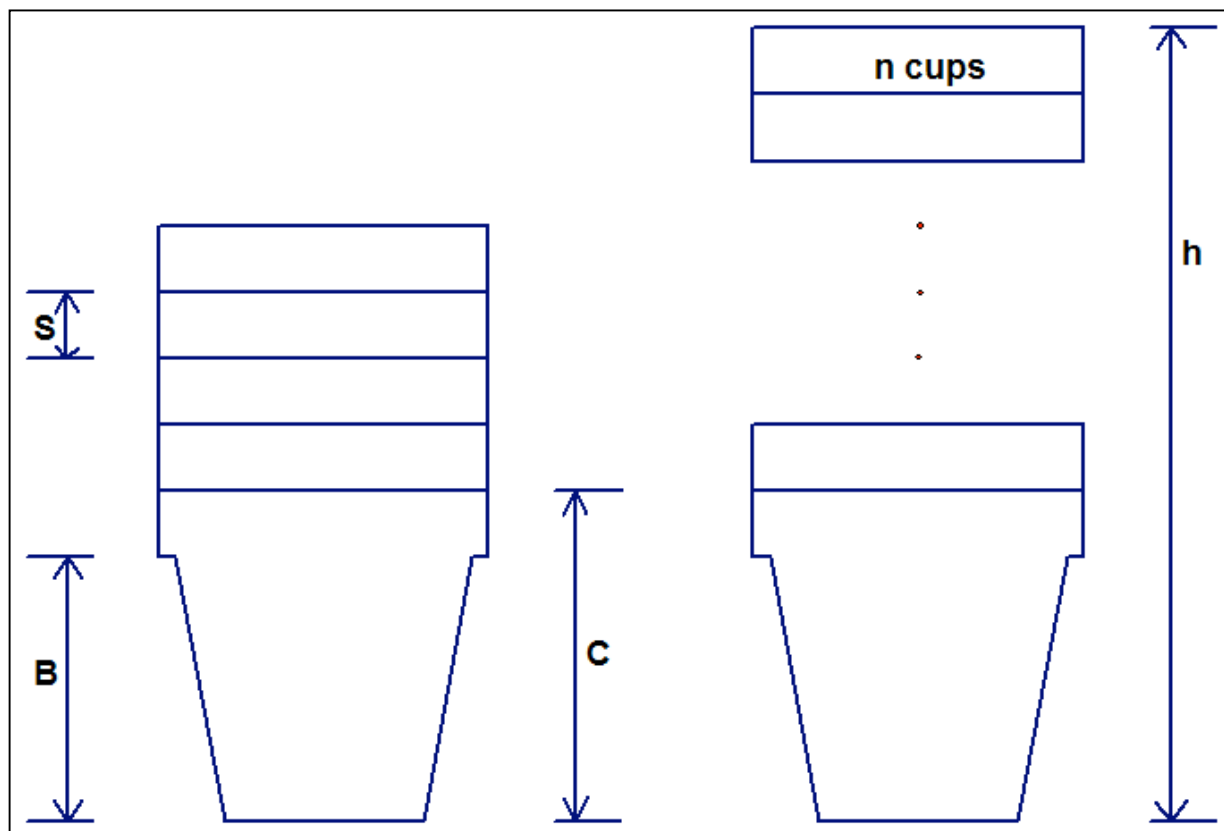
The dimensions of one particular type of cup have been labeled:

$C = 9.0$ cm = the individual cup height

$S = 1.5$ cm = the “stick up” height of the cup (see diagram for clarification)

$B = 7.5$ cm = the height of the individual cup without the stick up part

Figure 1



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3. Using the coordinate grid draw the **height function** graphs of some of these other cup types that have a (61, 99) stack. Include the extreme cases: the graph with greatest possible slope and a graph with a very small slope. All the graphs should include the point (61, 99).
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Part II:

5. In the questions in Part I, the height of the base of the cup (B) and the stick-up height of the cup (S) were changed while the number of the cups (n) and the total height (h) of the stack of cups were kept constant (61, 99). In part II, the height of the base of the cup (B) and the stick-up height of the cup (S) are held constant while the parameters n (the number of cups) and h (the height of the stack of cups) change.
- Starting with the function rule $h = 1.5n + 7.5$, find a new function rule for B in terms of S . Explain each step of your transformation in context.
 - What is the independent variable?
 - What is the dependent variable?
 - What is the domain of the problem situation?
 - What is the range of the problem situation?
6. a. On this new coordinate grid graph all the types of cup stacks such that 61 cups have a height of 99 cm.
- Is the graph of the problem situation continuous or not? Explain your thinking.
 - What are the x - and y -intercepts of the line? What are the meanings of the intercepts in the problem situation?
 - Is this relationship a direct variation or indirect variation? Explain.
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