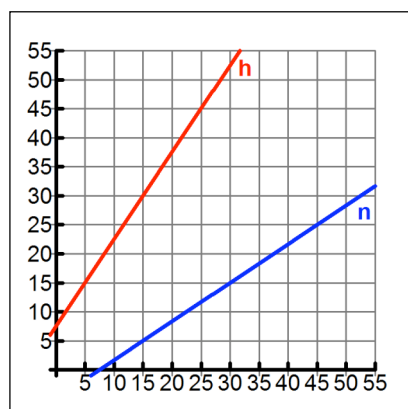
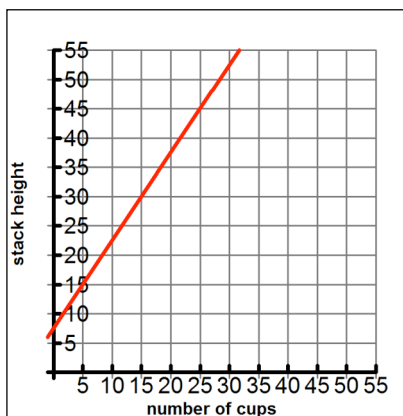
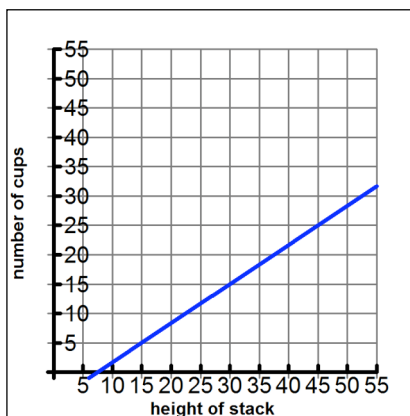


TASK 2.4.5: THE RELATIONSHIP OF THE HEIGHT FUNCTION AND THE COUNT FUNCTION
Solutions

The **height function**, $h = (1.5)n + 7.5$, and the **count function**, $n = \frac{1}{1.5}(h - 7.5)$, are closely related, where h is the height of the stack in centimeters and n is the cup count.

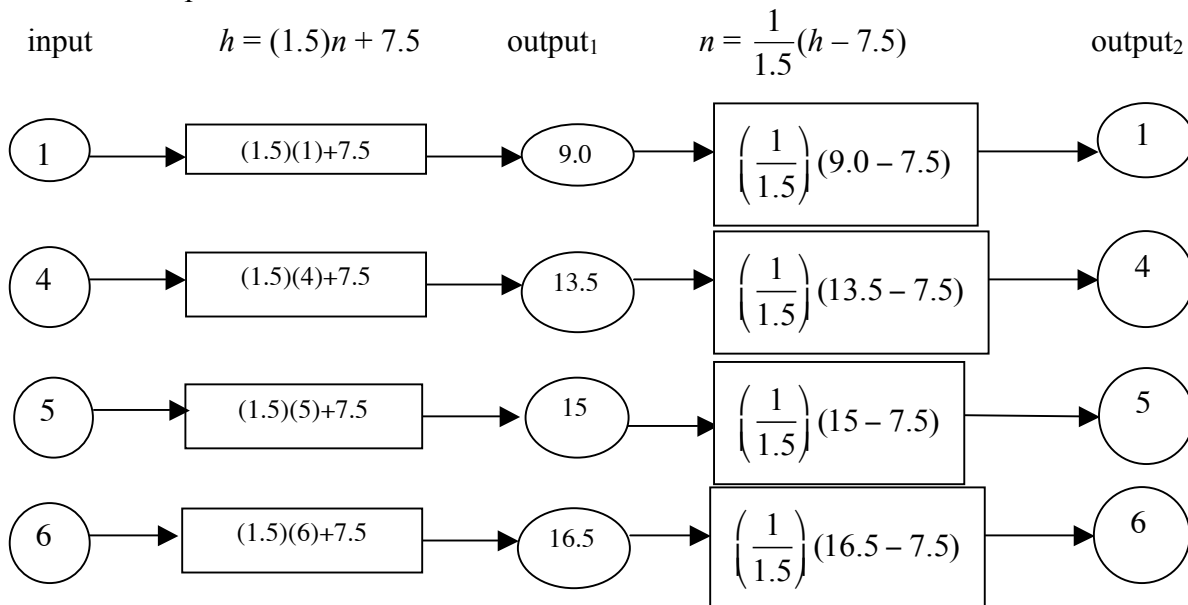
- Compare the graphs of $h = (1.5)n + 7.5$, the height function, to $n = \frac{1}{1.5}(h - 7.5)$, the count function.

For $h = (1.5)n + 7.5$, the independent variable is n , the number of cups in the stack, and the dependent variable is h , the height of the stack. For $n = \frac{1}{1.5}(h - 7.5)$, the independent variable is the height of the stack, h , and the dependent variable is the number of cups, n . If you draw these two graphs together on a dimensionless graph, one is the mirror image of the other across the $y = x$ line. (A MIRA or other reflector could be used to check if one relationship is the mirror image of the other.)



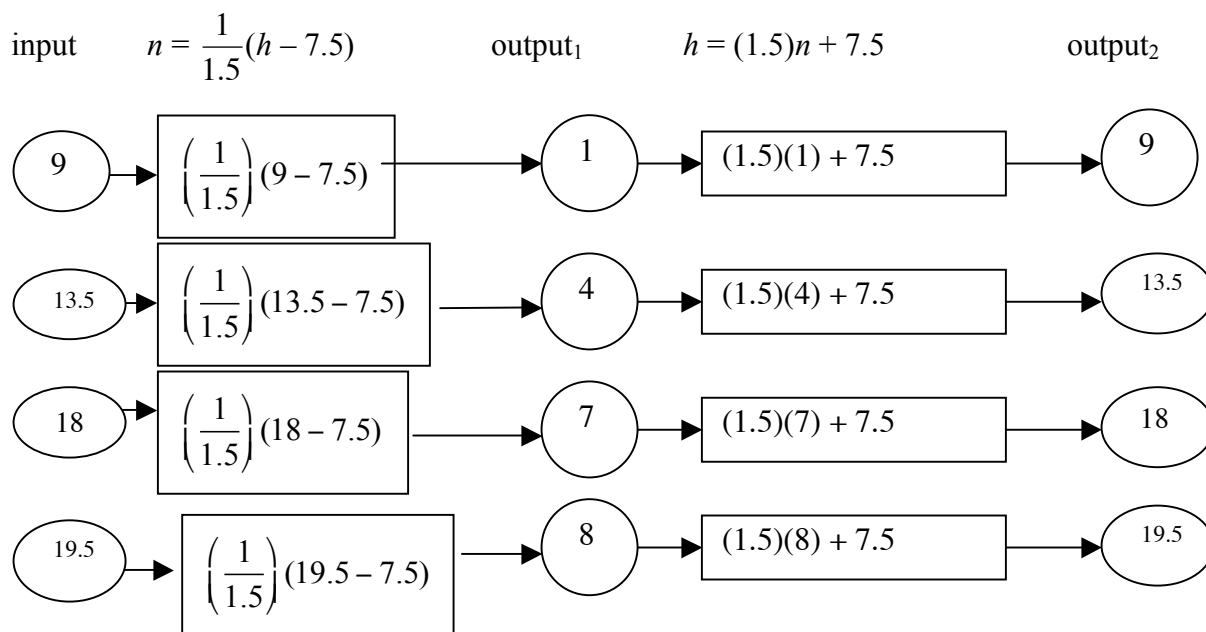
Algebra I: Strand 2. Linear Functions; Topic 4. Stack of Cups; Task 2.4.5

2. a. Complete the flow charts below.



b. What are the units for the input column above? For output₁? For output₂?
 # of cups; cm; # of cups

c. Complete the flow charts below



d. What are the units for the input column above? For output₁? For output₂?
 cm; # of cups; cm

Algebra I: Strand 2. Linear Functions; Topic 4. Stack of Cups; Task 2.4.5

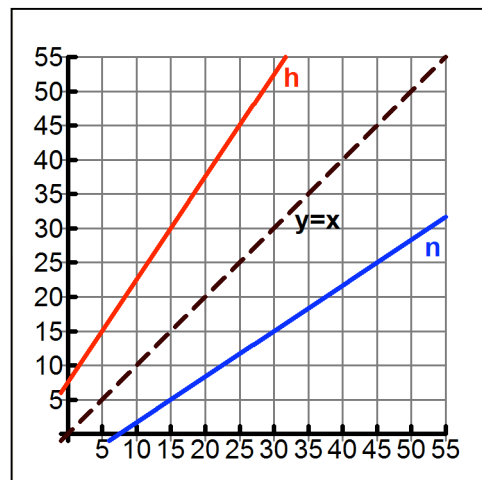
3. Write an explanation of what you see in the flow chart.

The output of the first function is the input of the second function. The result is that the output of the second function is the input of the first function.

This is observed regardless of which function is the first function (note 2a and 2b above).

4. If the height function were $y = f(x)$ and the count function is its inverse, $y = g(x)$, what would they look like graphed on the same coordinate grid with the parent function $y = x$? Compare these two graphs.

The parent function ($y = x$) is the line of symmetry, and the count function is the “mirror image” of the height function and vice versa. (Again, a MIRA or other reflector could be used to check if one relationship is the mirror image of the other.)



Math notes

This represents **doing** and **undoing**, which is investigated in middle school and in Algebra I. In formal mathematical terms, this process that the students investigate in middle school and in Algebra I is called inverse function. The process represented in the flow charts in 2a and 2b can be represented mathematically as $f(g(x))$ for 2a and $g(f(x))$ for 2b.

In problem #1, we graphed the number of cups as a function of the height of the stack (the **count function**) and the height of the stack as a function of the number of cups (the **height function**) on 2 different coordinate planes. These functions are inverses of each other; when we graph them on the same plane we see that one is a reflection of the other about $y = x$. Although both are measured in numbers, the axes of the resulting coordinate plane can no longer be interpreted in the context of the problem. The axes would represent two different quantities simultaneously; that is, the x- and y-axis would represent both number of cups as well as height of the stack in cm. This is an example of moving from a contextual situation to a mathematical model of the situation to a purely mathematical situation. Teachers may benefit from having conversations about these ideas.

TASK 2.4.5: THE RELATIONSHIP OF THE HEIGHT FUNCTION AND THE COUNT FUNCTION

In the stacks of cups problem, we are interested in the relationship between two variables h and n :

h = the height of the stack in cm

n = the number of cups in the stack

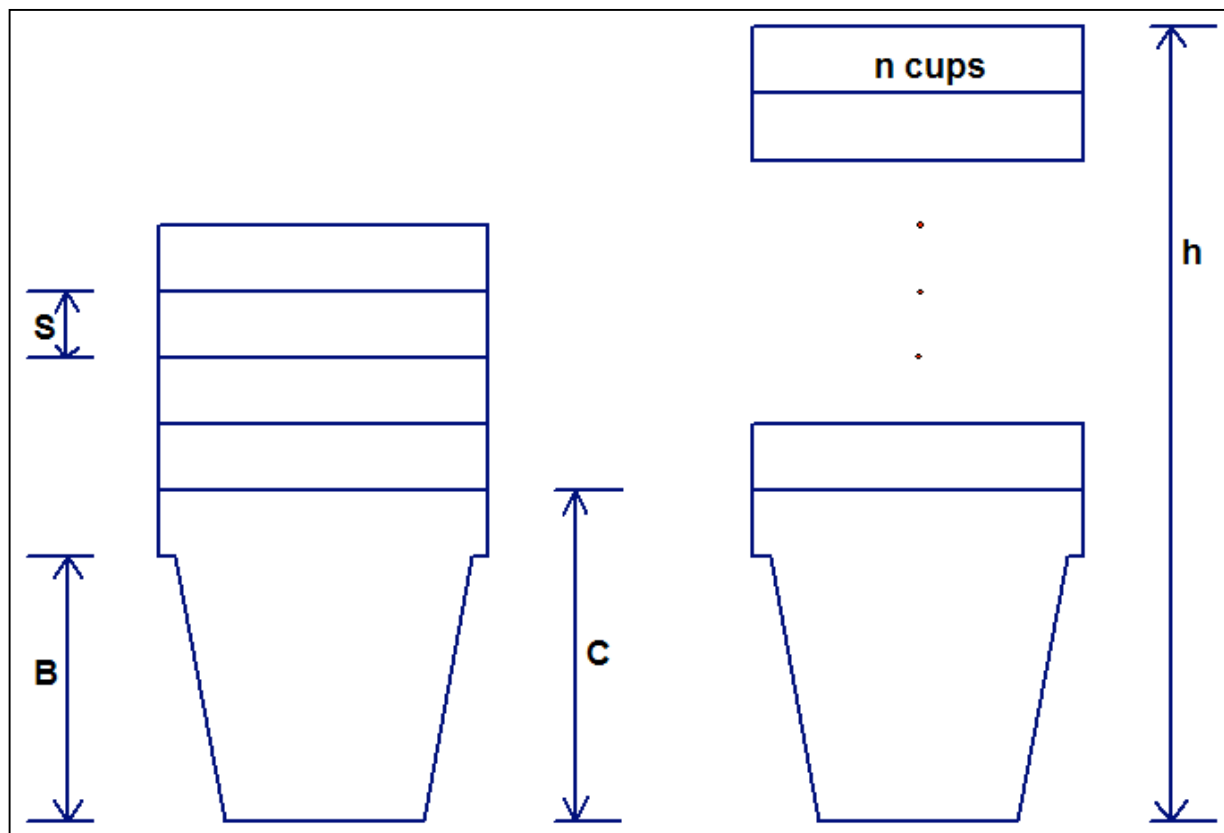
The dimensions of one particular type of cup have been labeled:

$C = 9.0$ cm = the individual cup height

$S = 1.5$ cm = the “stick up” height of the cup (see diagram for clarification)

$B = 7.5$ cm = the height of the individual cup without the stick up part

Figure I

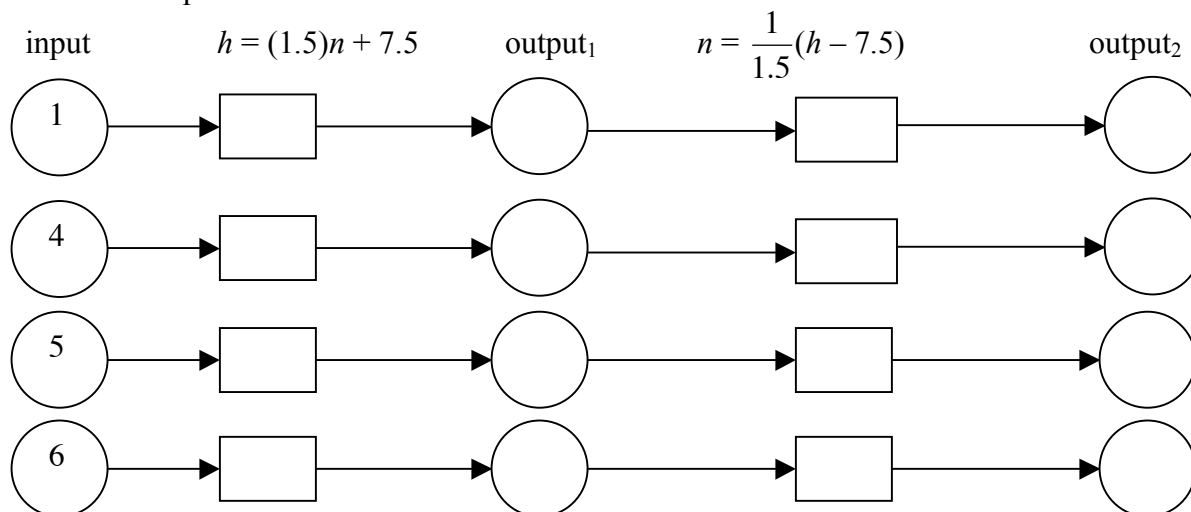


Algebra I: Strand 2. Linear Functions; Topic 4. Stack of Cups; Task 2.4.5

The height function, $h = (1.5)n + 7.5$, and the count function, $n = \frac{1}{1.5}(h - 7.5)$, are closely related, where h is the height of the stack in centimeters and n is the cup count..

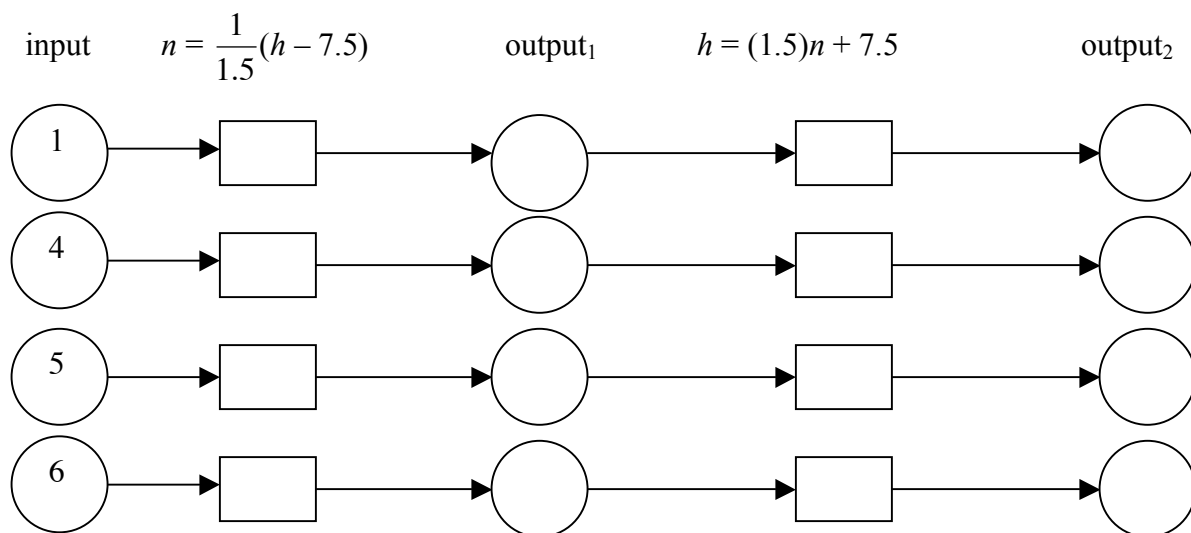
1. Compare the graphs of $h = (1.5)n + 7.5$, the height function, to $n = \frac{1}{1.5}(h - 7.5)$, the count function.

2. a. Complete the flow charts below.



b. What are the units for the input column? For output₁? For output₂?

c. Complete the flow charts below



d. What are the units for the input column? For output₁? For output₂?

Algebra I: Strand 2. Linear Functions; Topic 4. Stack of Cups; Task 2.4.5

3. Write an explanation of what you see in the flow chart.
4. If the height function were $y = f(x)$ and the count function is its inverse, $y = g(x)$, what would they look like graphed on the same coordinate grid with the parent function $y = x$. Compare these two graphs.