

**TASK 2.4.4: THE COUNT FUNCTION****Solutions**

Grocers stack containers on shelves daily. When grocers stack containers on shelves, the height of the shelves determines the number of containers that can be stacked. In this scenario, the **count function** gives the number of containers,  $n$ , in terms of the height of the stack. (This means that measuring the height of the stack and substituting this number into the count function gives the number of cups in the stack, so that the number of cups does not have to be counted directly.)

The following steps derive the **count function** from the height function,  $h = (1.5)n + 7.5$ , where  $h$  is in centimeters:

1. If you were given the height of a stack of cups, explain how you could find the number of cups in a stack. Use your explanation to determine the number of cups in a stack 46.5 cm tall.

Subtract the base of the cup, which in this case is 7.5 cm, from the height, then divide the difference by the measure of the stick-up of the cup, which in this case is 1.5 cm. The number of cups is the greatest integer less than or equal to the numerical answer.

$$n = \frac{46.5 - 7.5}{1.5}$$

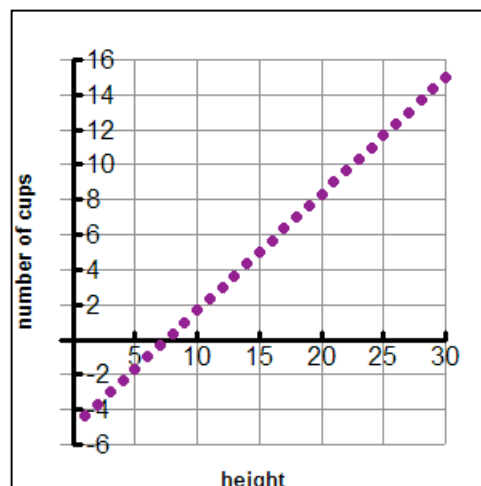
$$n = \frac{39}{1.5}$$

$$n = 26 \text{ cups}$$

2. Using your process in #1, write the number of cups in a stack as a function of the height of the stack of cups in cm.

$$n = \left( \frac{h - 7.5}{1.5} \right) \quad \text{or} \quad n = \left( \frac{1}{1.5} \right) (h - 7.5)$$

3. a. Use the list function of your graphing calculator to make a scatter plot for values of  $h$  from 1 to 30 cm. Label the axes appropriately. ( $x$  is height in cm;  $y$  is number of cups).

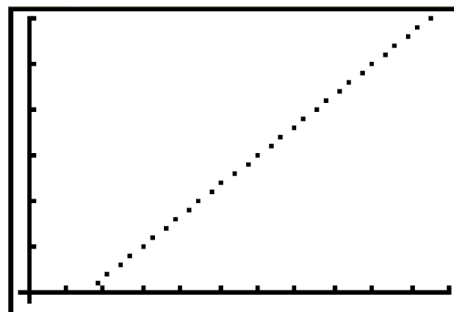


*Algebra I: Strand 2. Linear Functions; Topic 4. Stack of Cups; Task 2.4.4*

- b. Is the graph discrete or continuous? Explain. **Discrete; the independent variable increases the height of the stack-up, which in this case is 1.5 cm. Possible values for the dependent variable are 9.0 cm, 10.5 cm, 13.0 cm, 14.5 cm, etc.**
4. a. What is the independent variable? Is this  $L_1$  or  $L_2$  in your calculator? **The independent variable is the height of the stack of cups in cm. ( $L_1$ )**
- b. What is the dependent variable? Is this  $L_1$  or  $L_2$  in your calculator? **The dependent variable is the number of cups. ( $L_2$ )**
5. a. What domain makes sense for the problem situation?  
 **$\{7.5, 9.0, 10.5, 12.0, \dots\}$**
- b. What is the corresponding range?  **$\{1, 2, 3, 4, \dots\}$**
- c. Compare the domain and range of the **count function** problem situation to the domain and range of the **height function** problem situation.  
**The domain of the height function is the range of the count function.**

**The range of the height function is the domain of the count function.**

6. Change the window in your calculator and create a scatter plot of this count function.



7. a. What is the slope of the graph?  $\frac{1 \text{ cup}}{1.5 \text{ cm}}$
- b. What is the meaning of the slope of the graph in terms of the stack including the units?

**The slope is the rate of change in the number of cups for each 1.5 centimeter change in height.**

8. Consider the line through the discrete points of the scatter plot, that is the graph of the mathematical function. What is the “ $n$ -intercept” of the function graph? What does this meaning have in the problem situation? **- 5**

**None; it would be the number of cups for 0 height of the stack.**

9. What is the  $h$ -intercept of this graph? Explain what the  $h$ -intercept means in the problem situation.  
**7.5 cm. There is no physical meaning; it would be the height of 0 cups.**

**Math notes**

In Task #1 participants measured stacks of cups to find the height of the stacks, producing the function  $h = (1.5)n + 7.5$ , where measurements are given in centimeters. In Task #4, participants are given the height of the stacks of cups and they are asked to find the number of cups in the stack. **This habit of mathematical (algebraic) thinking is called doing and undoing.** One of the processes we want to help develop during this course and in our students is this notion of reversibility. That is, once students do something, many times they can work backwards and gain new insight, a more in-depth understanding of what they did, and an improved conceptual understanding of new concepts. Participants will recognize that the **count function** is the inverse of the **height function**.

### TASK 2.4.4: THE COUNT FUNCTION

In the stacks of cups problem, we are interested in the relationship between two variables  $h$  and  $n$ :

$h$  = the height of the stack in cm

$n$  = the number of cups in the stack

The dimensions of one particular type of cup have been labeled:

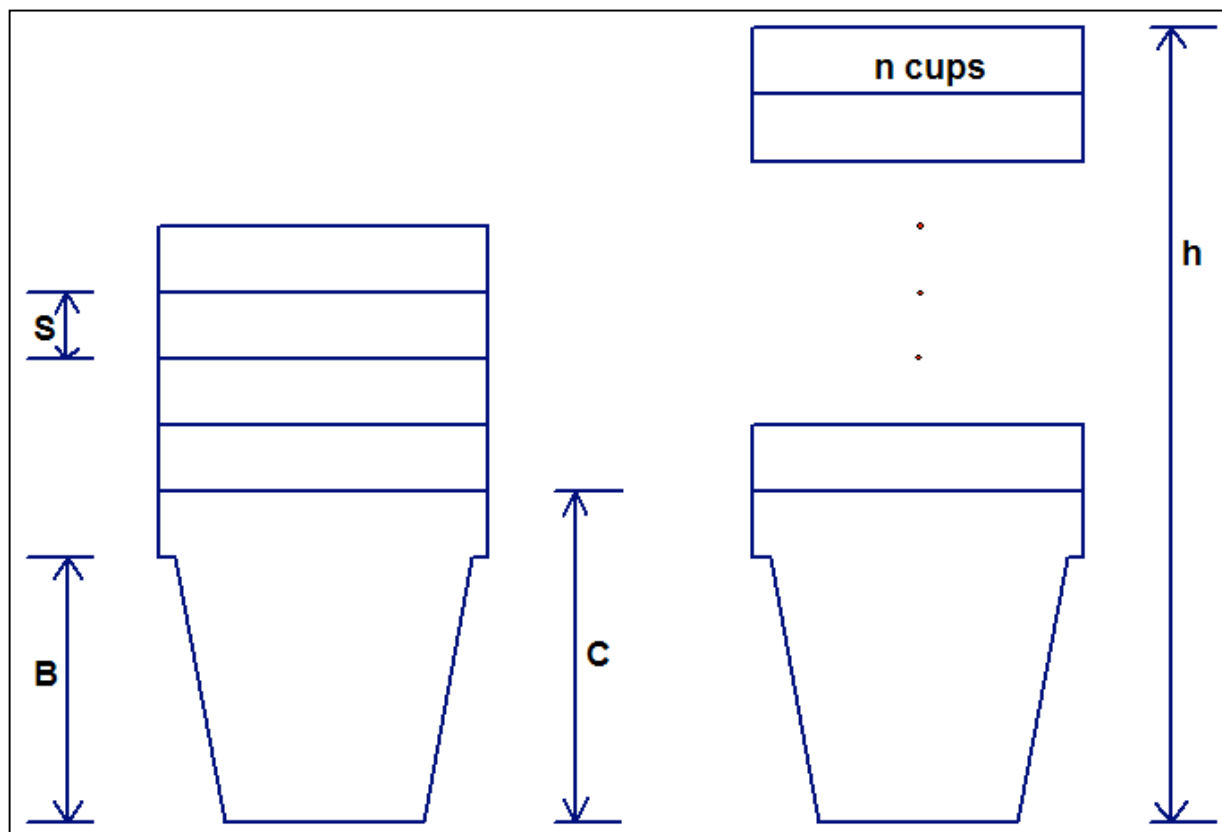
$C = 9.0$  cm = the individual cup height

$S = 1.5$  cm = the “stick up” height of the cup (see diagram for clarification)

$B = 7.5$  cm = the height of the individual cup without the stick up part

This question is about the **count function** for stacks of cups in Figure 1.

**Figure 1**



*Algebra I: Strand 2. Linear Functions; Topic 4. Stack of Cups; Task 2.4.4*

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The following steps derive the **count function** from the **height function**,  $h = (1.5)n + 7.5$ , where  $h$  is in centimeters:

1. If you were given the height of a stack of cups, explain how you could find the number of cups in a stack. Use your explanation to determine the number of cups in a stack 46.5 cm tall.
2. Using your process in #1, write the number of cups in a stack as a function of the height of the stack of cups.
3.
  - a. Use the list function of your graphing calculator to make a scatter plot for values of  $h$  from 1 to 30 cm. Label the axes appropriately.
  - b. Is the graph discrete or continuous? Explain.
4.
  - a. What is the independent variable? Is this  $L_1$  or  $L_2$  in your calculator?
  - b. What is the dependent variable? Is this  $L_1$  or  $L_2$  in your calculator?
5.
  - a. What domain makes sense for the problem situation?
  - b. What is the corresponding range?
  - c. Compare the domain and range of the **count function** problem situation to the domain and range of the **height function** problem situation.
6. Change the window in your calculator and create a scatter plot of this count function.
7.
  - a. What is the slope of the graph?
  - b. What is the meaning of the slope of the graph in terms of the stack including the units?

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8. Consider the line through the discrete points of the scatter plot, that is, the graph of the mathematical function. What is the “ $n$ -intercept” of the function graph? What meaning does this have in the problem situation?
9. What is the  $h$ -intercept of this graph? Explain what the  $h$ -intercept means in the problem situation.