

TASK 2.4.1: THE HEIGHT FUNCTION

Solutions:

In the stacks of cups problem, we are interested in the relationship between two variables h and n :

h = the height of the stack in cm

n = the number of cups in the stack

The dimensions of one particular type of cup have been labeled:

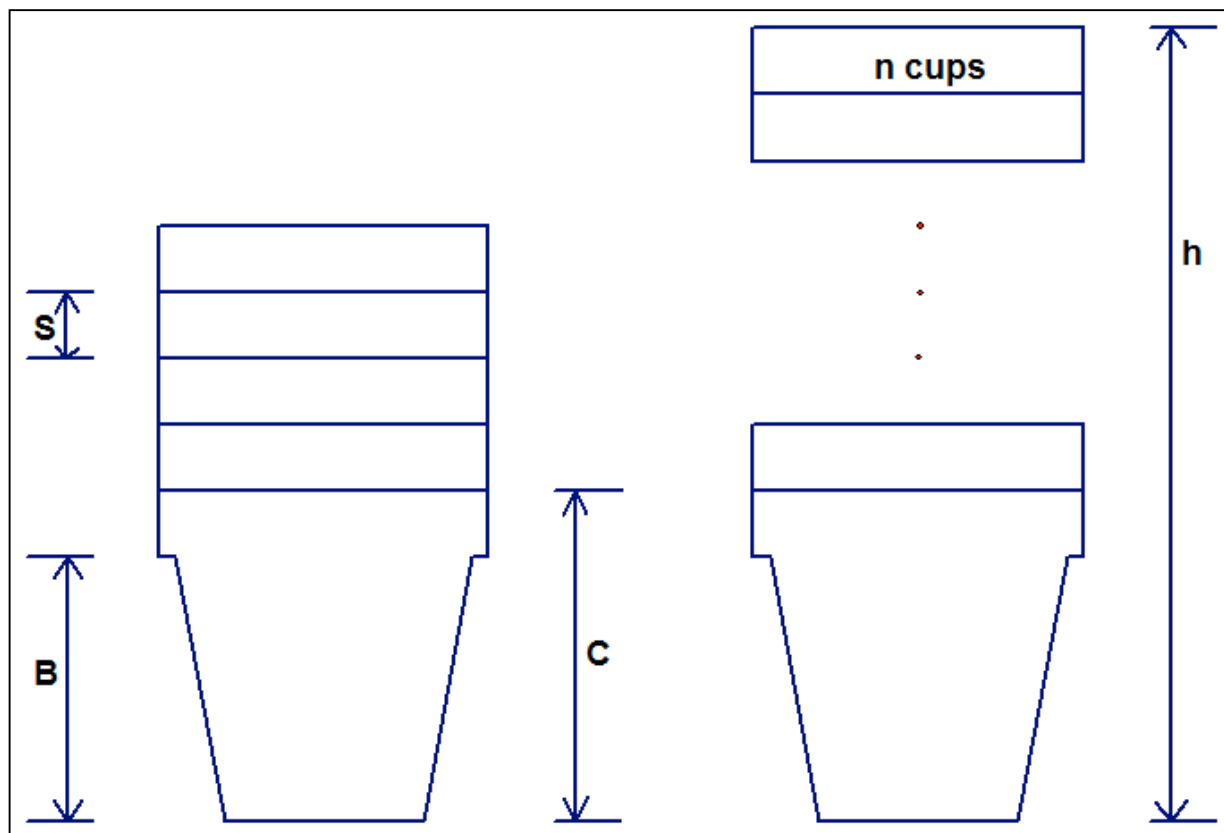
$C = 9.0$ cm = the individual cup height

$S = 1.5$ cm = the “stick up” height of the cup (see diagram for clarification)

$B = 7.5$ cm = the height of the individual cup without the stick up part

This question is about the **height function** for stacks of cups in Figure I.

Figure I



The Height Function

The **height function** gives a way of “measuring by counting.” (By this we mean that counting the number of cups in a stack and substituting this number into the height function gives the height of the stack, so that the height does not have to be measured directly.)

1. Estimate the height of a stack of 30 cups. **Students may have rulers available and measure one cup and one stick up part, then use computation to determine the height of 30 cups. They may estimate the height of one cup and one stick up part and then use computation, or they may estimate the height of one cup and one stick up part and then estimate the height of a stack of 30 cups. Participants may come up with other common sense methods to answer this question.**
2. Complete the table. Using the list function of a graphing calculator, input the number of cups in L_1 and the height of the stack in L_2 .

Table II

Number of Cups (L_1)	Process	Height of Stack (cm) (L_2)
1	$(1.5)(1) + 7.5$	9.0
2	$(1.5)(2) + 7.5$	10.5
3	$(1.5)(3) + 7.5$	12.0
4	$(1.5)(4) + 7.5$	13.5
5	$(1.5)(5) + 7.5$	15.0
...
n	$(1.5)n + 7.5$	$1.5n + 7.5$

L1	L2	L3
1	9	█
2	10.5	█
3	12	█
4	13.5	█
5	15	█
6	16.5	█
7	18	█
L3(1)=		

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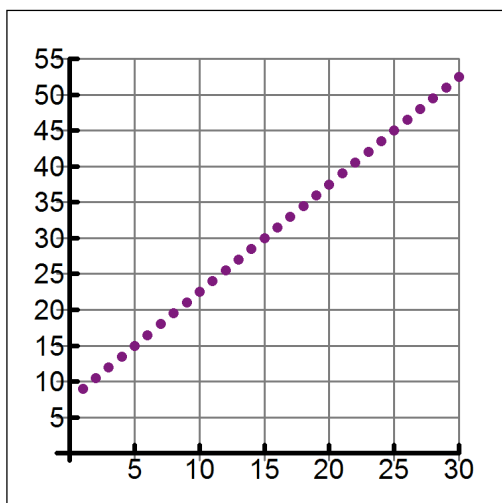
3. Write a formula for the height in cm of a stack of cups, h , as a function of the number of cups, n . This is the “height function” for this type of cup. Explain in words the meaning of this function. $h = (1.5)(n - 1) + 9.0$ or $h = (1.5)n + 7.5$

The height of one cup is 9 cm. The height of the stack increases 1.5 cm with the addition of each cup.

-- OR --

The height of one cup without the stick up part is 7.5 cm. For the stick-up part of each cup in the stack, the height of the stack is increased by 1.5 cm in the stack.

4. Using graph paper, create a scatter plot that represents the range and domain for this situation.
- a. Write a dependence statement that describes the situation.



The height of the stack is dependent on the number of cups.

- b. Label the axes appropriately.

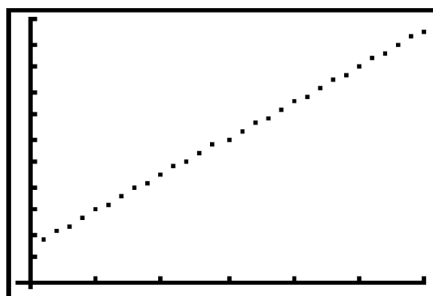
The horizontal axis is the independent variable, number of cups.

The vertical axis is the dependent variable, the height of the stack in cm.

- c. Is this graph discrete or continuous? Explain your choice.

Discrete; possible values for n are whole numbers that represent the number of cups.

5. Using your window and L_1 and L_2 , create a scatter plot of this data.



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6. The line through the points on the scatter plot is the graph of the mathematical function that models the situation.
- What is the “ h -intercept” of the linear function? 7.5 cm
 - What is the meaning of the h -intercept in terms of these cups? **The height of an individual cup without the stick up part.**
 - When there are no cups, the stack is 7.5 cm. Does this make sense? Why or why not? **Theoretically, when the number of cups is zero, there is no lip; only the base of the cup exists. Using real models, when the number of cups is zero, there is no base or lip and the height is 0. This makes no physical sense for our problem, hence the domain for the problem situation is integers greater than or equal to 1.**
 - What are the units of the h -intercept? **cm**
7. a. What is the slope of the line graph? $\frac{1.5 \text{ cm}}{1 \text{ cup}}$
- What is the meaning of the slope of the graph in terms of the stacks, including the units? **The slope is the rate of change for each cup added to the stack or the increase in the height of the stack of cups with the addition of each cup. The units of the slope are $\frac{\text{cm}}{\text{cup}}$**
8. a. Show how to find the height of a stack of 45 cups in three ways.
- graphing calculator
the maximum value of the independent value must be at least 45.
 - algebra
 $h = (1.5)(45 - 1) + 9.0 \Rightarrow h = (1.5)(44) + 9.0 \Rightarrow h = 66 + 9.0 = 75 \text{ cm}$
- or
- $$h = (1.5) 45 + 7.5 \Rightarrow h = 67.5 + 7.5 \Rightarrow h = 75 \text{ cm}$$
- graph
Starting at the origin on the graph, count over to 45 and go up to the graph of the function. The distance up is the height of the stack of cups in cm.
 - table
Extend the table to 45 cups.
- Will changing from 30 cups to 45 cups affect the range or domain? **It will affect the range and domain of the problem situation; the domain is now the integers $\{1, 2, 3, \dots, 45\}$**

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and the range is $\{9, 10.5, 12, \dots, 75\}$. You may need to reformat your graphing calculator window to see the new values.

c. How do you need to change the window on the graphing calculator to see this point? Increase the maximum value of the independent variable to at least 45 and the maximum value of the dependent variable to at least 75.

9. Consider the relationship between the number of cups and the height of stacks. Is this relationship proportional? Is it a direct variation? Justify your answer.

The relationship is not a proportion or a direct variation.

From the graph, the line does not include the origin.

From algebra, a nonzero constant, b , is added to the variable.

From the table, the ratios $\frac{n}{h}$ are not constant and do not equal $\frac{\Delta n}{\Delta h}$.

Math notes

Participants are asked to label the axes appropriately. An appropriate label for the horizontal axis of the height function is the dependent variable, number of cups, and appropriate label for the vertical axis is the independent variable, height of the stack in cm. Participants are also asked the unit of the horizontal axis and the vertical axis. The unit of the horizontal axis, the independent variable, is cups. The unit of the vertical axis, the dependent variable, is centimeters.

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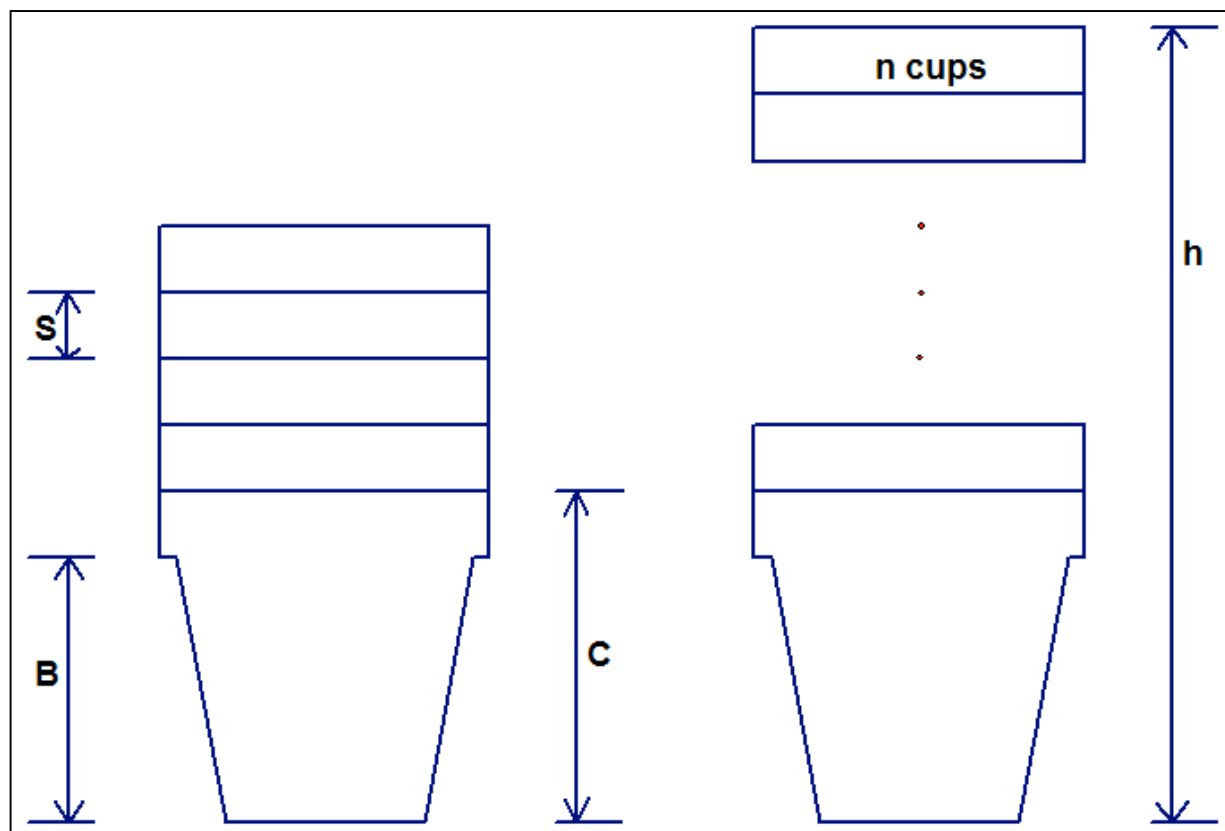
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Figure I



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Table II

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n		

3. Write a formula for the height in cm of a stack of cups, h , as a function of the number of cups, n . This is the “height function” for this type of cup. Explain in words the meaning of this function.

4.
 - a. Using graph paper, create a scatter plot of this problem situation for the values $n = 1$ to $n = 30$.
 - a. Write a dependence statement that describes the situation.

 - b. Label the axes appropriately.

 - c. Is this graph discrete or continuous? Explain your choice.

5. Using your window and L_1 and L_2 , create a scatter plot of this data.

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6. The line through the points on the scatter plot is the graph of the mathematical function that models the situation.
 - a. What is the “ h -intercept” of the linear function?
 - b. What is the meaning of the h -intercept in terms of these cups?
 - c. When there are no cups, the stack is 7.5 cm. Does this make sense? Why or why not?
 - d. What are the units of the h -intercept?
7.
 - a. What is the slope of the line graph?
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 - a. Show how to find the height of a stack of 45 cups in three ways.
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9. Consider the relationship between the number of cups and the height of stacks. Is this relationship proportional? Is it a direct variation? Justify your answer.