

## **STRAND 2: LINEAR FUNCTIONS**

### **TOPIC 2.3: $Y=MX$**

#### **Topic 2.3 notes**

##### **Mathematical focus**

Further development of linear functions requires investigation of the formula  $y = mx$ . Using contextual situations, participants investigate the effect of varying the coefficient of the independent variable,  $x$ . Participants translate among algebraic, tabular, graphical, and verbal representations of linear functions. They build an intuitive and formal understanding of slope as rate of change.

Terms: function, independent variable, dependent variable, domain, range, rate of change, constant rate,  $h$ -intercept, slope, proportional relationship, and direct variation

##### **Topic overview**

There are six tasks included in this topic:

Task 2.3.1: Formal Development of Slope

Task 2.3.2: The Height Function: Applying Slope

Task 2.3.3: Boxes of Hexagons: Solving Equations Based on the Height Function

Task 2.3.4: The Count Function

Task 2.3.5: The Relationship between the Height Function and the Count Function

Task 2.3.6: Paths between Graphs

In these activities, participants complete the activity with minimal instruction. After they complete each activity, participants share and discuss the activity in depth, especially the mathematics and the scaffolding questions required to guide students to an in-depth understanding. Participants collect, organize, and graph data. They investigate the effect of a non-unit coefficient of  $x$  on the graph of the function. As they investigate the function  $y = mx$ , they further develop their understanding of independent variable, dependent variable, domain, range, proportional, and direct variation. They translate from concrete models to tables, graphs, verbal descriptions, and equations.

In addition, participants consider a specific solution to a functional relationship, the inverse of a function, the relationship of a function and its inverse, and the meaning of the paths between the graph of a function and its inverse.

**TE<sub>x</sub>ES standards focus**

**TE<sub>x</sub>ES Standard II.004 Patterns and algebra.** The teacher uses patterns to model and solve problems and formulate conjectures. The beginning teacher:

- (A) Recognizes and extends patterns and relationships in data presented in tables, sequences, or graphs.

**TE<sub>x</sub>ES Standard II.005 Patterns and algebra.** The teacher understands attributes of functions, relations, and their graphs. The beginning teacher:

- (A) Understands when a relation is a function
- (B) Identifies the mathematical domain and range of functions and relations and determines reasonable domains for given situations.
- (C) Understands that a function represents a dependence of one quantity on another and can be represented in a variety of ways (e.g., concrete models, tables, graphs, diagrams, verbal descriptions, symbols).

**TE<sub>x</sub>ES Standard II.006 Patterns and algebra.** The teacher understands linear and quadratic functions, analyzes their algebraic and graphical properties, and uses them to model and solve problems. The beginning teacher:

- (A) Understands the concept of slope as a rate of change and interprets the meaning of slope and intercept in a variety of situations.

**TEKS/TAKS focus**

**TEKS A.1 Foundations for functions.** The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways. The student is expected to:

- (A) describe independent and dependent quantities in functional relationships;
- (B) gather and record data and use data sets to determine functional relationships between quantities;
- (C) describe functional relationships for given problem situations and write equations or inequalities to answer questions arising from the situations; and
- (D) represent relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities.

High School TAKS Objective 1: The student will describe functional relationships in a variety of ways.

**TEKS A.2 Foundations for functions.** The student uses the properties and attributes of functions. The student is expected to:

- (B) identify mathematical domains and ranges and determine reasonable domain and range values for given situations, both continuous and discrete.

High School TAKS Objective 2: The student will demonstrate an understanding of the properties and attributes of functions.

**TEKS A.5 Linear functions.** The student understands that linear functions can be represented in different ways and translates among their various representations.

The student is expected to:

- (A) determine whether or not given situations can be represented by linear functions; and

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(C) use, translate, and make connections among algebraic, tabular, graphical, or verbal descriptions of linear functions.

**TEKS A.6 Linear functions.** The student understands the meaning of the slope and intercepts of the graphs of linear functions and zeros of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student is expected to:

- (A) develop the concept of slope as rate of change and determine slopes from graphs, tables, and algebraic representations;
- (B) interpret the meaning of slope and intercepts in situations using data, symbolic representations, or graphs; and
- (D) graph and write equations of lines given characteristics such as two points, a point, and a slope, or a slope and  $y$ -intercept.

High School TAKS Objective 3: The student will demonstrate an understanding of linear functions.

## Materials

Task	#1	#2	#3	#4	#5	#6
Hexagons	x	x	x	x	x	
Handout	x	x	x	x	x	x
Rulers	x	x	x	x	x	
Graph Paper	x	x	x	x	x	
Graphing Calculator	x	x	x	x	x	

## Procedure

### Task 2.3.1

Arrange students in groups of 3 to 4. Distribute one copy of Stacks of Hexagons handout to each participant.

Participants arrange hexagons in stacks of two. These hexagons are wrapped in clear plastic packages and cannot be separated. Participants investigate the relationship of the number of pairs of hexagons and the height of the stacks. Ask participants to complete in groups the activity **Task 2.3.1 Formal Development of Slope**. Through this activity participants will formally develop the concept of slope as rate of change.

### Task 2.3.2

Participants arrange hexagons in stacks of two. Participants investigate the relationship of the number of pairs of hexagons and the height of the stacks. Ask participants to complete in groups the activity **Task 2.3.2 The Height Function**. This activity moves participants intuitively from a verbal description of a situation to a graphical representation of the situation. In addition, participants investigate independent variable, dependent variable, domain, range,  $h$ -intercept, rate of change, constant rate, slope, proportional, and direct variation. After most of the participants have completed the activity, share and discuss their findings.

**Task 2.3.3**

Provide time for participants to complete the activity **Task 2.3.3 Boxes of Hexagons: Solving Equations Based on the Height Function**. In this activity participants write and solve an equation and interpret the solution for a situation that limits the height of the stacks of hexagons. After most of the participants have completed the activity, share and discuss their findings.

**Task 2.3.4**

Provide time for participants to complete the activity **Task 2.3.4 The Count Function**. In this activity participants investigate the inverse of the function  $h = (2)n$ ; that is, for a given height,  $h$ , of the stack what is the number,  $n$ , of pairs of hexagons? After most of the participants have completed the activity, share and discuss their findings.

**Task 2.3.5**

Provide time for participants to complete the activity **Task 2.3.5 The Relationship of the Height Function and the Count Function**. In this activity, participants investigate the relationship of the height function and the count function from graphs of each on the same coordinate system. After most of the participants have completed the activity, share and discuss their findings.

**Task 2.3.6**

Provide time for participants to complete the activity **Task 2.3.6 Paths Between Graphs**. In this activity participants consider two different types of hexagons from graphical representation. From the graphical representation, the participants write verbal descriptions of various components of the graph. For example, what is the interpretation of the horizontal distance on the graph for the same ordinate and the vertical distance on the graph for the same abscissa? After most of the participants have completed the activity, share and discuss their findings.

**Summary**

After completing this activity, participants will have a better understanding of the tie between proportional situations and direct variation.

**Extensions**

Give students similar tasks varying the number of hexagons that are wrapped together. Ask questions such as,

- What is the new unit rate?
- How does the number of hexagons in a grouping affect the slope?
- Is your relationship proportional?

Provide participants with other proportional situations so that they can then look at them graphically, tabularly, and with the corresponding function. Refer to the TEXTEAMS institute *Proportionality Across the TEKS* for numerous activities involving proportionality.

**Assessments/Transition to the classroom**

Participants complete the Teacher's Journal, recording their responses and making any modifications necessary to make the tasks completely classroom ready.

