

TASK 2.3.4: THE COUNT FUNCTION**Solutions**

The “count function” for stacks of these groups of hexagons gives the number of pairs of hexagons n in terms of the height h of the stack.

The count function gives a way of “counting by measuring”. (This means that measuring the height of the stack and plugging this number into the count function gives the number of pairs of hexagons in the stack, so that the number of pairs of hexagons does not have to be counted directly.)

The following steps derive the count function from the height function of Task 2.3.2.

1. Given a height of 18 cm, how do you find the number of pairs of hexagon in a stack, using estimation or a method of measurement?

Participants may measure the height of a pair of hexagons and then use computation to determine the number of pairs of hexagons. They may estimate the height of one pair and then use computation to determine number of pairs of hexagons. Participants may come up with other common sense methods to answer this question. (These stacks of hexagons are “shrink wrapped” together and sold as one unit. Pairs cannot be separated.)

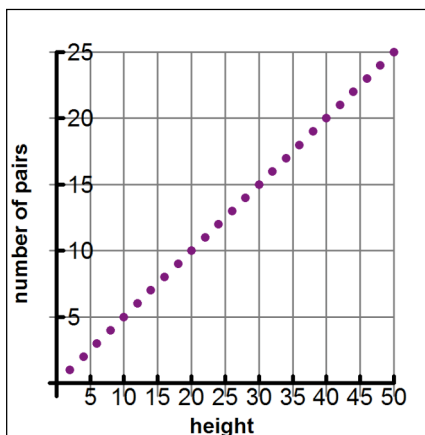
2. Complete Table III.

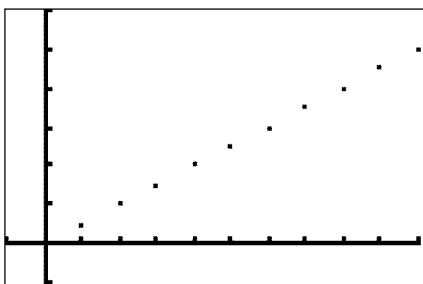
Table III
Stacks of Hexagons

Height of Stacks (cm)	Process	Number of Pairs
2	$\left(\frac{1}{2}\right) (2)$	1
4	$\left(\frac{1}{2}\right) (4)$	2
6	$\left(\frac{1}{2}\right) (6)$	3
8	$\left(\frac{1}{2}\right) (8)$	4
10	$\left(\frac{1}{2}\right) (10)$	5
...
h	$\left(\frac{1}{2}\right) (h)$	$\left(\frac{1}{2}\right) h$

* These data are based on hexagon blocks that are 1 cm high. Participants should complete the table using the measurements of hexagon blocks they are using.

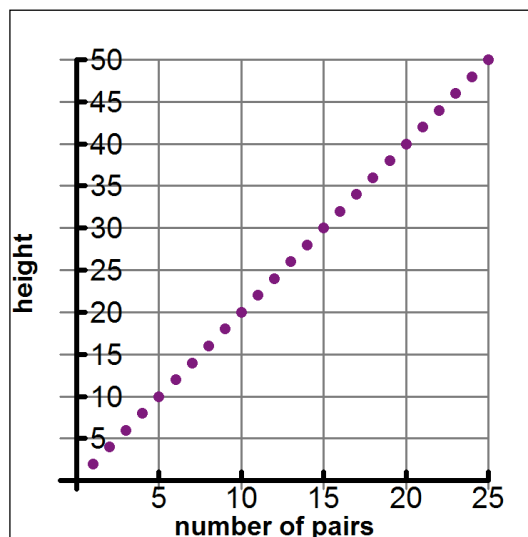
3. Using the coordinate grid, graph this function for values of h from 0 to 50 cm with h as the independent variable and n as the dependent variable. This time, the number of pairs is dependent on the height of the stack. Now enter your table into your calculator and create a scatter plot of your data. What is the range and domain for this situation? How does knowing the range and domain of the modeling function help to set the window on the graphing calculator?





Setting the graphing calculator window depends on the data you want to enter and examine. The domain of the data for the problem situation is the set of even natural numbers (with no upper limit, theoretically). However, the h -values of interest in this problem end at 50. Therefore, a suggested viewing window would have a minimum x -value not less than 2, and the maximum x -value not greater than 50. The suggested minimum y -value for the window is not less than 1 and the maximum y -value for the window is not greater than 25. In some cases the domain and the x -values in the viewing window may be the same; however, the viewing window may be set differently depending on what part of the graph is of interest. However, some educators recommend setting the minimum value 10% less than the domain or range; and the maximum value 10% greater than the domain or range to enhance the viewing area.

4. Here is a copy of the height scatter plot that we created earlier. Compare the range and domain of this scatter plot (the height function) to the scatter plot in #3 (the count function). Compare the tables of the two scatter plots. Compare the rules for the data for each scatter plot. Write a paragraph that explains the changes that you observed. *Answers will vary but participants should notice that the domain of the height function is the range of the count function and vice versa.*



5. Create the graph of the function that models the data by entering your rule into $Y_1=$.
6. a. What is the slope of the count function?

$$\frac{1}{2\text{cm}}$$

- b. What is the meaning of the slope of the linear model in terms of the stack, including the units?

The slope is the rate of change in the number of pairs for each 2 centimeters change in height. The units of the slope are pair/centimeter.

7. What is the meaning of the point (0, 0) on the linear model?
For 0 height, there are 0 pairs of hexagons.

8. Does this relationship represent direct variation? Explain and support your answer with the graph, table, and equation.

Yes

The graph includes the origin and the rate of change is constant, specifically $\frac{1}{2}$.

The rate of change, $\frac{\Delta y}{\Delta x} = \frac{y}{x} = \frac{1}{2}$ and includes the pair with height of 0 cm and number of pairs 0.

The equation is $h = \frac{1}{2}n$. b is 0, that is, 0 is the constant added to the variable.

Math notes

In Task 2.3.2, participants stacked pairs of hexagons and found the height of the stacks. In Task 2.3.4, participants are given the height of the stacks and find the number of pairs of hexagons in the stack. This habit of mathematical (algebraic) thinking is called doing and undoing. One of the processes we want to help develop during this course and in our participants is this notion of reversibility. That is, once participants do something, oftentimes they can work backwards and gain new insight, a more in-depth understanding of what they did, and an improved conceptual understanding of new concepts. Participants will recognize that the count function is the inverse of the height function.

TASK 2.3.4: THE COUNT FUNCTION

For this activity, we are assuming that two hexagons are wrapped in clear plastic packages and cannot be separated. Simulate these packages by placing two yellow hexagon-shaped pattern blocks in groups of two and stack them one pair at a time.

We are investigating the relationship between the height (h) of the stack of hexagons and the number (n) of pairs of hexagons as represented in Figure I.

n = the number of pairs of hexagons in the stack

h = the height of the stack

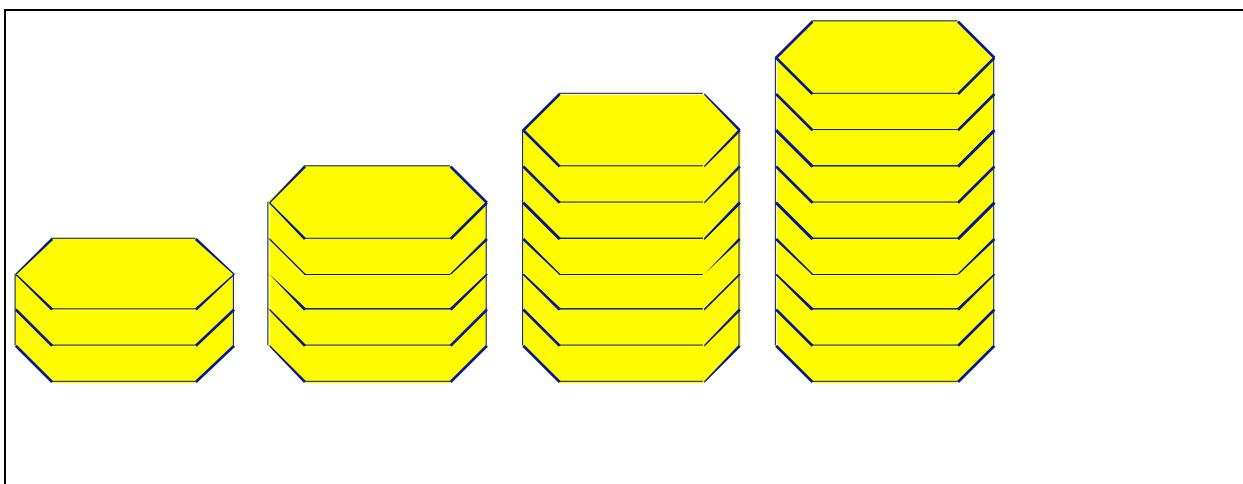


Figure I

The “count function” for stacks of these groups of hexagons gives the number of pairs of hexagons n in terms of the height h of the stack.

The count function gives a way of “counting by measuring”. (This means that measuring the height of the stack and plugging this number into the count function gives the number of pairs of hexagons in the stack, so that the number of pairs of hexagons does not have to be counted directly.)

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2. Complete Table III.

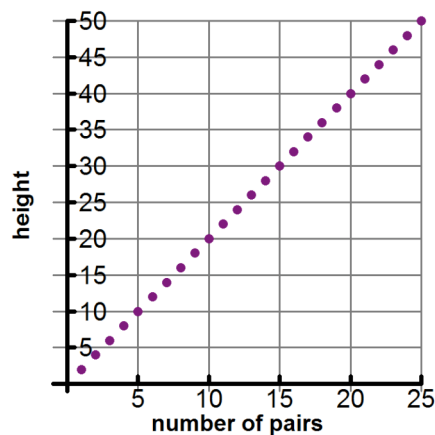
Table III
Stacks of Hexagons

Height of Stacks (cm)	Process	Number of Pairs
2		
4		
6		
8		
10		
...
h		

3. Using the coordinate grid, graph this function for values of h from 0 to 30 cm with h as the independent variable and n as the dependent variable. This time, the number of pairs is dependent on the height of the stack. Now enter your table into your calculator and create a scatter plot of your data. What is the range and domain for this situation? How does knowing the range and domain of the modeling function help to set the window on the graphing calculator?

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4. Here is a copy of the height scatter plot that we created earlier. Compare the range and domain of this scatter plot (the height function) to the scatter plot in #3 (the count function). Compare the tables of the two scatter plots. Compare the rules for the data for each scatter plot. Write a paragraph that explains the changes that you observed.



5. Create the graph of the function that models the data by entering your rule into $Y_1=$.
6. a. What is the slope of the count function?
- b. What is the meaning of the slope of the linear model in terms of the stack, including the units?
7. What is the meaning of the point (0, 0) on the linear model?
8. Does this relationship represent direct variation? Explain and support your answer with the graph, table, and equation.