

TASK 2.3.2: THE HEIGHT FUNCTION — APPLYING SLOPE**Solutions**

The height function gives a way of “measuring by counting.” (This means that counting the number of pairs of hexagons in a stack and plugging this number into the height function gives the height of the stack, so that the height does not have to be measured directly.)

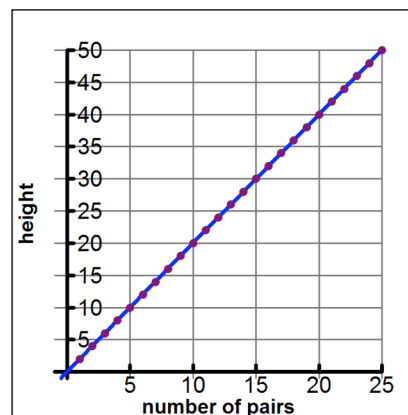
In this exercise, you will be using the lists and scatter plots that you created in the previous activity.

- Write a formula for the height h of a stack of hexagons as a function of the number n of pairs of hexagons. This is the “height function” which is measured in centimeters for this type of hexagon.

$$h = (2)n$$

- Using your graphing calculator, enter your formula into $Y_1=$. Graph this function and your scatter plot data. How are these alike? How are they different?

The formula, or function rule, that models the situation, graphs through the scatter plot data connecting the discrete points.



- The mathematical function that models this situation is a line through the discrete points of the scatter plot. What is the “ h -intercept” of the linear function?

$$h=0$$

- What is the meaning of the h -intercept in terms of these hexagons?
The height of zero pairs of hexagons is 0 cm.

- What is the unit of the h -intercept?
cm

4.

a. What is the slope of the graph of the function?

 2 cm/pair

b. What is the meaning of the slope of the graph of the function in terms of the stacks including the units?

*rate of change for each pair of hexagon added to the stack; the increase in the height (cm) of the stack of hexagons with the addition of each pair of hexagons (pair)**Slope units are $\frac{\text{cm}}{\text{pair}}$.*

5. How many pairs of hexagons does it take to make a stack 50 cm high?

 25 pairs 6. Using your graphing calculator, compute in L_3 , the ratio $\frac{\Delta \text{List}(L_2)}{\Delta \text{List}(L_1)} \left(\frac{\Delta \text{height}}{\Delta \text{number}} \right)$.*See Technology Notes for assistance in entering this into the calculator.*7. Using your graphing calculator, compute in L_4 , the ratio $\frac{L_2}{L_1} \left(\frac{\text{height}}{\text{number}} \right)$.8. What do you notice about $L_3 (= \frac{\Delta y}{\Delta x})$ and $L_4 (= \frac{y}{x})$? *L_3 and L_4 should be the same.*

9. Is the relationship between the number of pairs and the height a proportional relationship? Is this relationship a direct variation? Justify your answer.

yes; yes; constant of proportionality, $k = \frac{\Delta y}{\Delta x} = \frac{y}{x} = \text{slope of the line, } m$ *Note: $L_4 (= \frac{y}{x})$ is the constant of proportionality from a proportional relationship. When* *$\frac{y}{x}$ is the same as the slope, $\frac{\Delta y}{\Delta x}$, the relationship is a direct variation. A direct variation is a functional representation of a proportional situation.*

10. Given that the relationship in Table II is proportional, complete Table II.

Table II
Stacks of Hexagons

Number of Pairs (L_1)	Process	Height of Stack (L_2) (cm)
1	$3(1)$	3
2	$3(2)$	6
3	$3(3)$	9
4	$3(4)$	12
5	$3(5)$	15
...
n	$3(n)$	$3n$

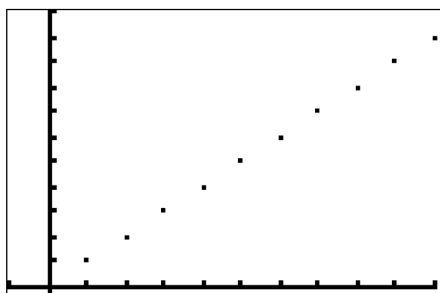
11. a. What is the constant of proportionality, k , using data in Table II.

$\frac{3}{1}$ or 3

- b. How does this constant of proportionality compare to the slope of the line?

constant of proportionality, $\frac{3}{1}$ or 3 is the same as the slope, 3

12. a. Using your graphing calculator, turn off the plot of the first data and create the scatter plot for the data in Table II .



- b. Enter your algebraic rule from Table II in $Y_1=$ in the graphing calculator. What is the y-intercept? What does the y-intercept mean in this situation? What are the units of the slope?

0; The height of zero (0) pairs of hexagons is 0.

Math notes

For any direct variation, the y-intercept is always 0, so the slope m is $\frac{\Delta y}{\Delta x} = \frac{y}{x}$ which is the constant of proportionality k . This is also the unit rate.

In the middle grades, students investigate proportionality and find that for a relationship to be proportional, two criteria are required: constant rate of change and graph includes the origin. In Algebra I, proportionality is related to direct variation and participants should be made aware that these ideas are virtually the same.

The relationship between the number of pairs of hexagons and the height of the stacks is a proportional relationship. Investigation of proportional relationships is a “big idea” in the middle grades. The ratio is a constant value that connects key concepts of unit rate, scale factor, and rate of change that can be used to find all possible values of the dependent variable.

Technology notes

Here are instructions for entering the $\frac{\Delta List(L_2)}{\Delta List(L_1)}$ lists into your calculator.

- Enter your data into L1 and L2.
- Arrow over to L3 and highlight L3 to give a command to the whole list.
- Press 2nd **STAT** to access the list menu.
- Arrow over to the OPS menu and arrow down to (7) $\Delta List($

- Press **ENTER**

- We want $\Delta List(L_2)/$
- Press 2nd **STAT** to access lists
- Press 2 (for L2)
- Press /

- ... $\Delta List(L_1)$
- Press 2nd **STAT** to access the list menu.
- Arrow over to the OPS menu and arrow down to (7) $\Delta List($
- Press 2nd **STAT** to access lists
- Press 2 (for L2)

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- Press **ENTER**

L1	L2	L3	3
1	2	3	
4	6	9	
8	10	12	
12	14		

L3(1)=2			

- Arrow over and highlight L4
- Type in L2/L1

L2	L3	L4	4
2			
4			
6			
8			
10			
12			
14			

L4=L2/L1			

- Press **ENTER**

L2	L3	L4	4
2		2	
4		4	
6		6	
8		8	
10		10	
12		12	
14		14	

L4(1)=2			

TASK 2.3.2: THE HEIGHT FUNCTION — APPLYING SLOPE

For this activity, we are assuming that two hexagons are wrapped in clear plastic packages and cannot be separated. Simulate these packages by placing two yellow hexagon-shaped pattern blocks in groups of two and stack them one pair at a time.

We are investigating the relationship between the height (h) of the stack of hexagons and the number (n) of pairs of hexagons as represented in Figure I.

n = the number of pairs of hexagons in the stack

h = the height of the stack

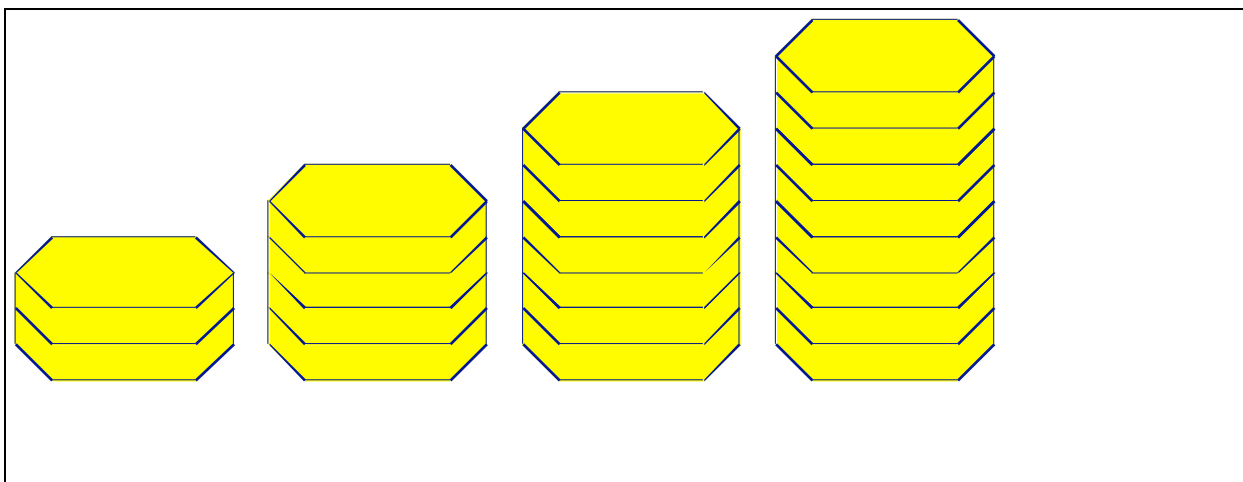


Figure I

The height function gives a way of “measuring by counting.” (This means that counting the number of pairs of hexagons in a stack and plugging this number into the height function gives the height of the stack, so that the height does not have to be measured directly.)

In this exercise, you will be using the lists and scatter plots that you created in the previous activity.

1. Write a formula for the height h of a stack of hexagons as a function of the number n of pairs of hexagons. This is the “height function” which is measured in centimeters for this type of hexagon.
2. Using your graphing calculator, enter your formula into $Y_1=$. Graph this function and your scatter plot data. How are these alike? How are they different?

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3.
 - a. The mathematical function that models this situation graphs as a line through the discrete points of the scatter plot. What is the “ h -intercept” of the linear function?
 - b. What is the meaning of the h -intercept in terms of these hexagons?
 - c. What is the unit of the h -intercept?
4.
 - a. What is the slope of the graph of the function?
 - b. What is the meaning of the slope of the graph of the function in terms of the stacks including the units?
5. How many pairs of hexagons does it take to make a stack 50 cm high?
6. Using your graphing calculator, compute in L_3 , the ratio $\frac{\Delta List(L_2)}{\Delta List(L_1)}$, $\left(\frac{\Delta height}{\Delta number} \right)$.
7. Using your graphing calculator, compute in L_4 , the ratio $\frac{L_2}{L_1}$, $\left(\frac{height}{number} \right)$.
8. What do you notice about $L_3 (= \frac{\Delta y}{\Delta x})$ and $L_4 (= \frac{y}{x})$?
9. Is the relationship between the number of pairs and the height a proportional relationship? Is this relationship a direct variation? Justify your answer.

Algebra I: Strand 2. Linear Functions; Topic 3. $Y=MX$; Task 2.3.2

10. Given that the relationship in Table II is proportional, complete Table II.

Table II
Stacks of Hexagons

Number of Pairs (L_1)	Process	Height of Stack (L_2) (cm)
1		
2		
3		
4		
5		15
...
n		

11. a. What is the constant of proportionality, k , using data in Table II.
- b. How does this constant of proportionality compare to the slope of the line?
12. a. Using your graphing calculator, turn off the plot of the first data and create the scatter plot for the data in Table II.
- b. Enter your algebraic rule from Table II in $Y_1=$ in the graphing calculator. What is the y -intercept? What does the y -intercept mean in this situation? What are the units of the slope?