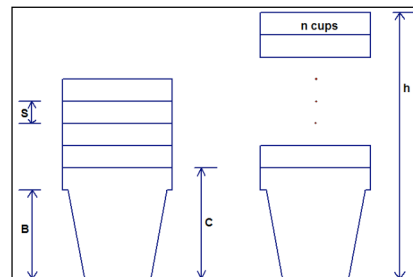


TASK 2.2.1: CUPS, CUPS, CUPS**Solutions**

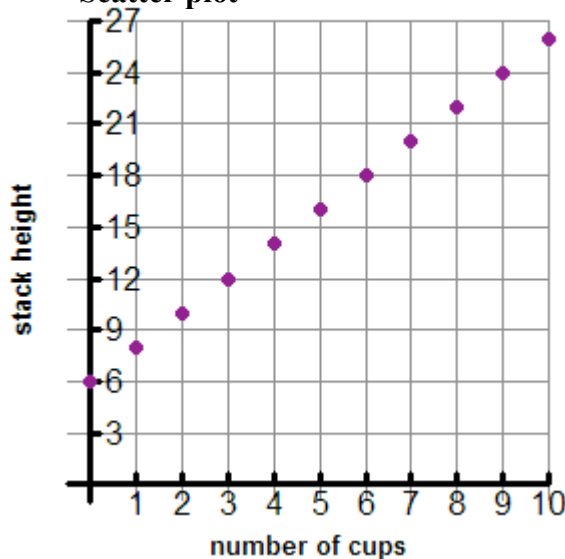
James, Maryanne, Bill, and Debbie work at Texas Cup Company. They are each assigned to package different sizes of cups for shipment in boxes. For each of the following problem situations, fill in the table, make a scatter plot, and write the function rule that models this situation. Be sure to label the graph and the table.



- James is stacking cups in boxes 40 cm high, where the stack is increasing at a rate of 2 cm per cup. James did not measure the height of the first cup, but he knows the height for the stack is 14 cm after 4 cups are stacked.

Table

Number of Cups	Height
0	6 cm
1	8 cm
2	10 cm
3	12 cm
4	14 cm
...	...
n	$2n + 6$

Scatter plot

Function rule: $h = 2n + 6$

- What is the domain and range for this situation?
Domain = {0,1,2,..., 17}; Range = {6, 8, 10,..., 40}
- When the independent variable is 0, what is the corresponding value for the dependent variable? **6 cm**
- What does this ordered pair mean in the mathematical model? What does it mean in the context of the cup-stacking situation? Is this realistic or not?
The ordered pair (0,6) is where the function crosses the y-axis.
In the cup-stacking situation, it is the height of 0 "stick-ups"; the height of the base without the "stick-up" portion.

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No, this is not realistic; cups are not sold without the “stick-up” portion.

- d. The point (0,6) is called the **y-intercept**, why do you think it was given this name? **The function intersects the y-axis at the point (0,6).**
- e. How did the constant change in height, $2 \frac{\text{cm}}{\text{cup}}$, allow you to create the table?
Starting with a height of 14 cm for 4 cups, subtract 2 cm repeatedly from the height for each cup less than 4, and add 2 cm to the height for each cup over 4.
- f. Explain how you see this constant rate of change in the scatter plot.
To move from one point to the next point, move 1 cup horizontally and 2 cm vertically to reach the next point.
- g. The mathematical term for this constant rate of change is **slope**. Using your rule, identify the slope and the y-intercept. **The slope is $2 \frac{\text{cm}}{\text{cup}}$. The y-intercept is 6 cm.**
- h. Write a sentence to describe the mathematical rule in the context of the situation.
When cups with a base of 6 cm and a “stick-up” height of 2 cm are stacked, the height of the stack increases 2 cm with the addition of each cup.
- i. Predict the height of a stack of 10 cups. **$h = 2n + 6 = 2(10) + 6 = 26$; height is 26 cm**
- j. If a stack of cups is 34 cm high, then how many cups are in the stack? **14 cups**

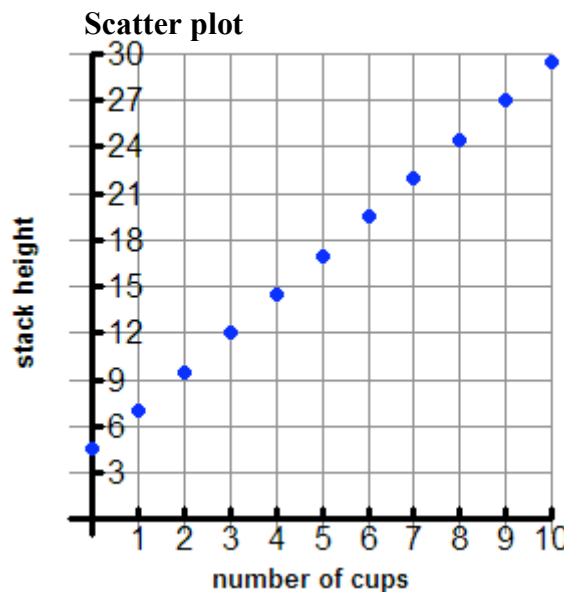
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2. Maryanne is creating a stack of cups 42 cm high, where the stack is increasing at a rate of 2.5 cm per cup. Maryanne did not measure the height of the first cup, but she knows the height of the stack is 24.5 cm after 8 cups are stacked.

Table

Number of Cups	Height
0	4.5 cm
2	9.5 cm
4	14.5 cm
6	19.5 cm
8	24.5 cm
...	...
n	$2.5n + 4.5$

Diagrammatic annotations: On the left, four brackets labeled '2' indicate the height increase between rows 0-2, 2-4, 4-6, and 6-8. On the right, four brackets labeled '5' indicate the height increase between rows 0-2, 2-4, 4-6, and 6-8.



Function rule: $h = 2.5n + 4.5$

- a. Are the domain and range the same as in (1)? If not, then how did they change and why? **The domain is not the same as in (1). The domain = {0,1,2,...,15} and the range = {4.5, 7, 9.5, ..., 42}. The rule changed and the height of the box changed which caused the domain and range to change.**
- b. When the independent variable is 0, what is the corresponding value for the dependent variable? **4.5 cm**
- c. What does this ordered pair mean in the mathematical model? What does it mean in the context of the cup stacking situation? Is this realistic or not?
The ordered pair (0, 4.5) is where the function crosses the y-axis. In the stacking cup situation, it is the height of 0 “stick-ups”, that is the height of the base without the “stickup” portion.
- No, this is not realistic; cups are not sold without the “stickup” portion.**
- d. What is the y-intercept? **(0, 4.5)**
- e. How did the constant change in height, $2.5 \frac{\text{cm}}{\text{cup}}$, allow you to create the table?

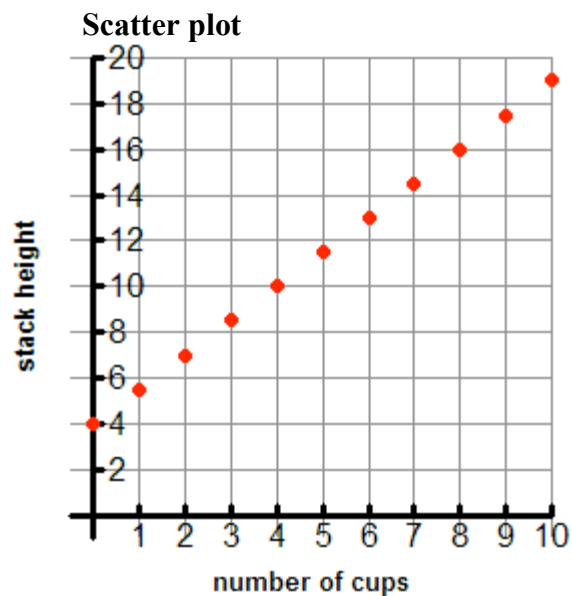
Starting with a height of 24.5 cm for 8 cups, subtract 2.5 cm repeatedly from the height for each cup less than 8 or subtract 5.0 cm repeatedly from the height for each set of 2 cups less than 8 cups.

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- f. Explain how you see this constant rate of change in the scatter plot. To move from one point to the next point, move 1 cup horizontally and 2.5 cm vertically to reach the next point, or move 2 cups horizontally and 5 cm vertically to reach the next point on the scatter plot.
- g. The mathematical term for constant rate of change is **slope**. Using your rule, identify the slope and the y-intercept. The slope is $2.5 \frac{\text{cm}}{\text{cup}}$. The y-intercept is 4.5 cm.
- h. Write a sentence to describe the mathematical rule. When cups with a base of 4.5 cm and a “stickup” height of 2.5 cm are stacked, the height of the stack increases 2.5 cm with the addition of each cup.
- i. Predict the height of a stack of 15 cups.
 $h = 2.5n + 4.5 = 2.5(15) + 4.5 = 37.5 + 4.5 = 42.0$
- j. If a stack of cups is 34.5 cm high, then how many cups are in the stack?
 12 cups
3. For the cups he was assigned, Bill noticed the height of 5 cups was 11.5 cm high and a stack of 8 cups was 16 cm high. Bill was packaging his cups in boxes 40 cm high.

Table

	Number of Cups	Height	
	0	4.0 cm	
1	1	5.5 cm	1.5
2	3	8.5 cm	3
2	5	11.5cm	3
1	6	13.0 cm	1.5
2	8	16.0 cm	3
	
	n	$1.5n + 4$	



Function rule: $h = 1.5n + 4$

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- a. When the independent variable is 0, what is the corresponding value for the dependent variable? **4.0 cm**
- b. What does this ordered pair mean in the mathematical model? What does it mean in the context of the cup-stacking situation? Is this realistic or not?
The ordered pair (0,4) is where the function crosses the y-axis. In the cup-stacking situation, it is the height of 0 “stick-ups”, the height of the base without the “stick-up” portion.

No, this is not realistic; cups are not sold without the “stick-up” portion.

- c. What is the constant change in height per cup added to the stack? **1.5 cm**
- d. Write a sentence to describe this function in words.
When cups with a base of 4.0 cm and a “stickup” height of 1.5 cm are stacked, the height of the stack increases 1.5 cm with the addition of each cup.
- e. Predict the height of a stack of 20 cups. **$h = 1.5n + 4 = 1.5(20) + 4 = 34$; height is 34 cm.**
- f. The mathematical term for constant rate of change is **slope**. Identify and explain the slope and y-intercept using the table and graph.

Table:

Finite differences can be used here to determine that the slope is $1.5 \frac{\text{cm}}{\text{cup}}$.

The y-intercept is the value on the table that is associated with the input value of 0.

Traditional Function Rule:

The slope is $\frac{8.5 - 5.5}{3 - 1} = \frac{3}{2} = 1.5 \frac{\text{cm}}{\text{cup}}$.

The y-intercept is the value of the dependent variable that corresponds to 0 for the independent variable. The y-intercept is 4 cm.

Graph:

The slope is the ratio from one point to a second point of the vertical distance to the horizontal distance. The y-intercept is the point where the function intersects the y-axis.

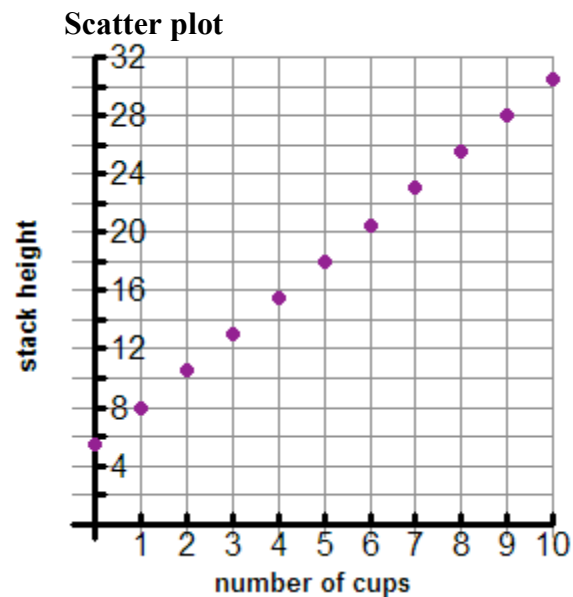
g. If a stack of cups is 22 cm high, then how many cups are in the stack? 12

4. Debbie is stacking cups in boxes, each 41 cm high, for shipment. The height of the cup without the “stick-up” part is 5.5 cm. She measures the height of a stack of 5 cups. The height is 18 cm.

Table

Number of Cups	Height
0	5.5cm
1	8.0 cm
2	10.5 cm
3	13.0 cm
4	15.5 cm
5	18.0 cm
...	...
n	$2.5n + 5.5$

Diagram illustrating the relationship between the number of cups and the height of the stack. The table shows that for each additional cup, the height increases by 2.5 cm. The y-intercept is 5.5 cm, representing the height of the base without the “stick-up” part.



Function rule: $h = 2.5n + 5.5$

a. What is the domain and range for this situation? Explain. Domain= {0, 1, 2, ..., 14}; Range={5.5, 8, 10.5, ..., 40.5}; If you stack more than 14 cups, the height is more than 40.5 cm. The range is the corresponding values to the domain values.

b. What does the ordered pair (0, 5.5) mean in the mathematical model? What does it mean in the context of the cup-stacking situation? Is this realistic or not?

The ordered pair (0, 5.5) is where the function crosses the y-axis.

In the stacking cup situation, it is the height of 0 “stick-ups”, that is the height of the base without the “stick-up”.

No, this is not realistic; base of the cups are not sold without the “stick-up”

c. For each additional cup, what is the change in the height? 2.5 cm

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What is the mathematical term for this value? **slope**

- d. How does the graph reflect the change in height? **To move from one point to the next point, move 1 cup horizontally and 2.5 cm vertically.**
- e. Write a sentence to describe the mathematical rule in the context of this situation. **When cups with a base of 5.5 cm and a “stick-up” height of 2.5 cm are stacked, the height of the stack increases 2.5 cm with the addition of each cup.**
- f. Predict the height of a stack of 9 cups.

$$h = 2.5n + 5.5 = 2.5(9) + 5.5 = 22.5 + 5.5 = 28.0$$

- g. The mathematical term for constant rate of change is **slope**. Identify and explain the slope and y -intercept using the table and graph.

Table:

Finite differences can be used here to determine that the slope is $2.5 \frac{\text{cm}}{\text{cup}}$.

The y -intercept is the value on the table that is associated with the input value of 0.

Traditional Function Rule:

The slope is the ratio of the change in the dependent variable for the corresponding change in the independent variable.

$$\text{The slope is } \frac{15.5 - 13.0}{4 - 3} = \frac{2.5}{1} = 2.5 \frac{\text{cm}}{\text{cup}}.$$

The y -intercept is the value of the dependent variable that corresponds to 0 for the independent variable. The y -intercept is 5.5 cm.

Graph:

The slope is the ratio from one point to a second point, or the vertical distance to the horizontal distance. The y -intercept is the point where the function intersects the y -axis.

- h. If a stack of cups is 23 cm high, then how many cups are in the stack? **7 cups**

Teaching notes

The 4 problems presented here each have a different focus.

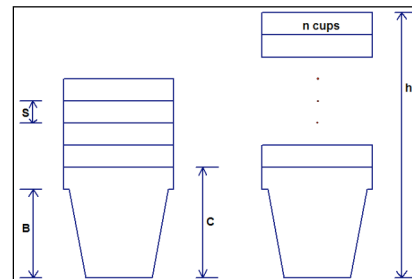
- The first provides a rate of change and requires students to complete the table. The x values start with 0 and increase by 1.
- The second problem has x values changing by 2's in the table and a rate of change of 2.5.

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- The third problem gives the student 2 coordinates and they must compute the rate of change and apply that information to complete the table.
- The fourth problem provides the y -intercept and another point on the table. Students must compute the rate of change and complete the table.

TASK 2.2.1: CUPS, CUPS, CUPS

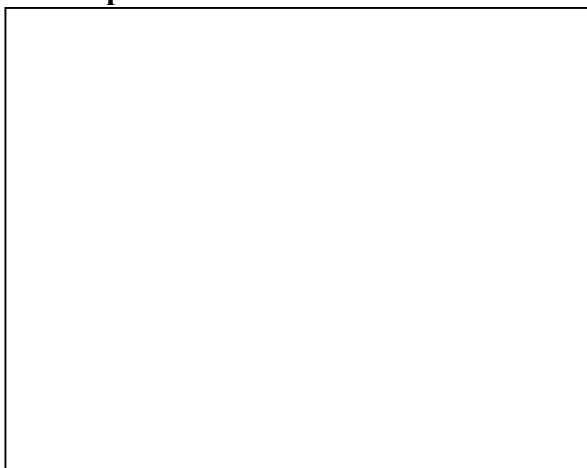
James, Maryanne, Bill, and Debbie work at Texas Cup Company. They are each assigned to package different sizes of cups for shipment in boxes. For each of the following problem situations, label the table and graph. Fill in the table, make a scatter plot, and write the function that models this situation.



- James is stacking cups in boxes 40 cm high, where the stack is increasing at a rate of 2 cm per cup. James did not measure the height of the first cup, but he knows the height for the stack is 14 cm after 4 cups are stacked.

Table

0	
1	
2	
3	
4	14 cm
...	...
n	

Scatter plot**Function rule:**

- What is the domain and range for this situation?
- When the independent variable is 0, what is the corresponding value for the dependent variable?
- What does this ordered pair mean in the mathematical model? What does it mean in the context of the cup stacking situation? Is this realistic or not?
- The point (0,6) is called the **y-intercept**, why do you think it was given this name?

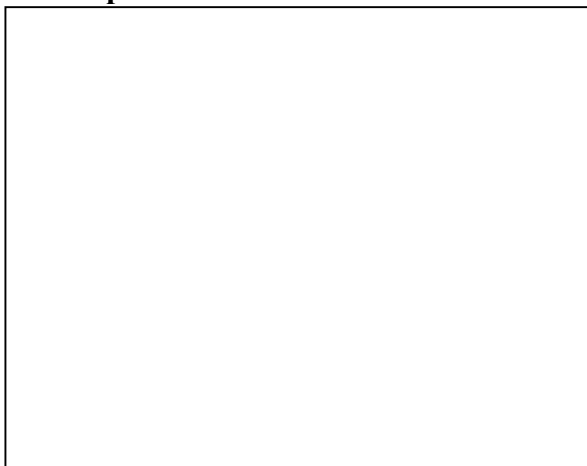
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- e. How did the constant change in height, $2 \frac{\text{cm}}{\text{cup}}$, allow you to create the table?
- f. Explain how you see this constant rate of change in the scatter plot.
- g. The mathematical term for this constant rate of change is **slope**. Using your rule, identify the slope and the y -intercept.
- h. Write a sentence to describe the mathematical rule in the context of the situation.
- i. Predict the height of a stack of 10 cups.
- j. If a stack of cups is 34 cm high, then how many cups are in the stack?
2. Maryanne is creating a stack of cups 42 cm high, where the stack is increasing at a rate of 2.5 cm per cup. Maryanne did not measure the height of the first cup, but she knows the height of the stack is 24.5 cm after 8 cups are stacked.

Table

0	
2	
4	
6	
8	24.5 cm
...	...
n	

Scatter plot



Function rule:

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- a. Are the domain and range the same as in (1)? If not, then how did they change and why?
- b. When the independent variable is 0, what is the corresponding value for the dependent variable?
- c. What does this ordered pair mean in the mathematical model? What does it mean in the context of the cup stacking situation? Is this realistic or not?
- d. What is the y -intercept?
- e. How did the constant change in height, $2.5 \frac{cm}{cup}$, allow you to create the table?
- f. Explain how you see this constant rate of change in the scatter plot.
- g. The mathematical term for constant rate of change is **slope**. Using your rule, identify the slope and the y -intercept.
- h. Write a sentence to describe the mathematical rule.
- i. Predict the height of a stack of 15 cups.
- j. If a stack of cups is 34.5 cm high, then how many cups are in the stack?

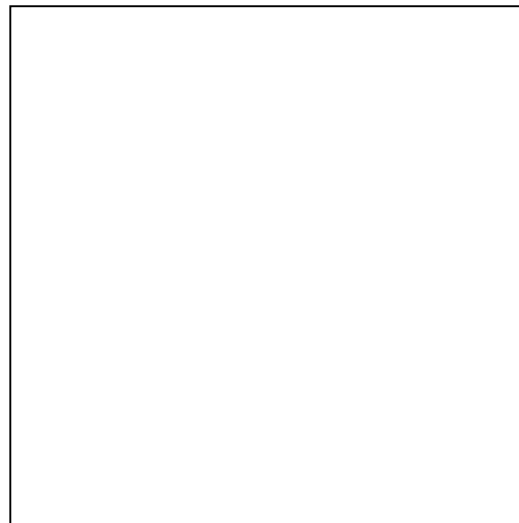
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3. For the cups he was assigned, Bill noticed the height of 5 cups was 11.5 cm. high and a stack of 8 cups was 16 cm high. Bill was packaging his cups in boxes 40 cm high.

Table

0	
1	
3	
5	11.5 cm
6	
8	16 cm
...	...
n	

Scatter plot



Function rule:

- When the independent variable is 0, what is the corresponding value for the dependent variable?
- What does this ordered pair mean in the mathematical model? What does it mean in the context of the cup stacking situation? Is this realistic or not?
- What is the constant change in height per cup added to the stack?
- Write a sentence to describe this function in words.
- Predict the height of a stack of 20 cups.
- The mathematical term for constant rate of change is **slope**. Identify and explain the slope and y -intercept using the table and graph.

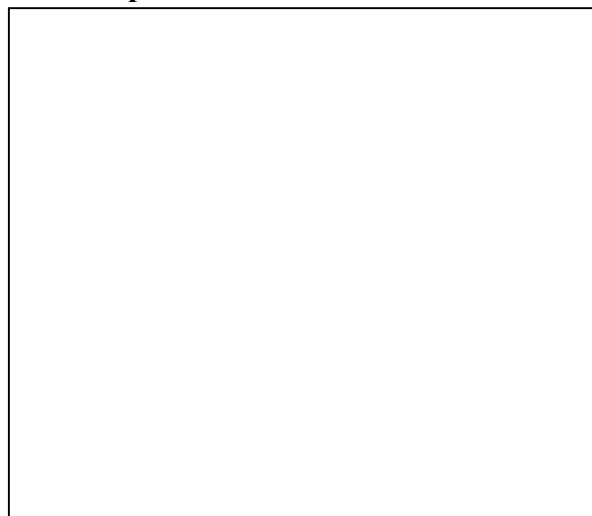
Algebra I: Strand 2. Linear Functions; Topic 2. Formalizing Slope and y-Intercept; Task 2.2.1

- g. If a stack of cups is 22 cm high, how many cups are in the stack?
4. Debbie is stacking cups in boxes, each 41 cm high, for shipment. The height of the cup without the “stick-up” part is 5.5 cm. She measures the height of a stack of 5 cups. The height is 18 cm.

Table

0	
1	
2	
3	
4	
5	18.0 cm
...	...
n	

Scatter plot



Function rule:

- What is the domain and range for this situation? Explain.
- What does the ordered pair $(0, 5.5)$ mean in the mathematical model? What does it mean in the context of the cup-stacking situation? Is this realistic or not?
- For each additional cup, what is the change in the height? What is the mathematical term for this value?
- How does the graph reflect the change in height?
- Write a sentence to describe the mathematical rule in the context of this situation.

