

TASK 2.10.1: WHAT'S THE CHANCE? — EQUATIONS**Solutions**

1. If the numbers 2, 5, and 7 randomly replace the letters a , b , and c without repetition in the equation $ax + b = c$, what is the probability that x will be negative? Justify your answer.

The probability is 1 out of 2. Many students will list all 6 possibilities:

$$2x + 5 = 7 (+)$$

$$2x + 7 = 5 (-)$$

$$5x + 2 = 7 (+)$$

$$5x + 7 = 2 (-)$$

$$7x + 5 = 2 (-)$$

$$7x + 2 = 7 (+)$$

Another way: solve for x : $x = (c-b)/a$ then substitute numbers

$$(2 - 5)/7 (-) \quad (2 - 7)/5 (-) \quad (5 - 7)/2 (-)$$

$$(5 - 2)/7 (+) \quad (7 - 2)/5 (+) \quad (7 - 5)/2 (+)$$

The big idea here is that only 2 of the three numbers affect the outcome, b and c .

If c is greater than b the answer will be positive, and if c is less than b the answer will be negative.

2. How many possible combinations of equations are there using the numbers 2, 5, and 7 to replace the letters a , b , and c in the equation $ax + b = c$, if repetition is allowed?

If repetition is allowed there will be 27 possible results. $27 = (3)(3)(3)$.

3. If the numbers in question #1 are changed to -2 , -5 , and -7 , then randomly replace the letters a , b , and c without repetition in the equation $ax + b = c$, what is the probability that x will be negative? Justify your answer.

All possibilities can be listed

$$-2x - 5 = -7 (+)$$

$$-2x - 7 = -5 (-)$$

$$-5x - 2 = -7 (+)$$

$$-5x - 7 = -2 (-)$$

$$-7x - 5 = -2 (-)$$

$$-7x - 2 = -7 (+)$$

The result did not change from question #1, there is still a 1 out of 2 chance for x to be negative. The same principle applies whether the numbers are negative or positive. If the absolute value of c is greater than the absolute value of b , the answer will be negative, but dividing by a negative number, a , means that x has a positive value. If the absolute value of c is less than the absolute value of b , the result will be positive, but dividing by a negative, a , will produce a negative value.

4. Would the probability that x will be negative change if a 4th number were added to the sample space, without repetition? Justify your answer.

No, only three numbers will be used at a time and it does not matter if the numbers are positive, negative, or a combination. The probability remains 1 out of 2. For any one triad of numbers, one-half of the equations will have c less than b , and thus will have a negative answer.

5. If the sample space includes the integers -100 to 100 , excluding 0 , without repetition, will the probability that x is negative change? Justify your answer. *No matter what numbers are included in the sample space, the probability remains 1 out of 2. $(c - b)$ will produce either a positive or negative number. Dividing by a will not change the probability of $1/2$ that x is negative.*

6. If the numbers 2 , 4 , and 6 randomly replace the letters a , b , and c in the formula $ax + by = c$, without repetition, what is the probability that the slope will be negative? What is the probability the slope will be positive? Justify your answer. The possible combinations are:

$$\begin{array}{lll} 2x + 4y = 6 \text{ (-m)} & 2x + 6y = 4 \text{ (-m)} & 4x + 2y = 6 \text{ (-m)} \\ 4x + 6y = 2 \text{ (-m)} & 6x + 2y = 4 \text{ (-m)} & 6x + 4y = 2 \text{ (-m)} \end{array}$$

When all the values are positive, each of the possible slopes will be negative. Only 2 values need to be considered, a and b . The probability then of the slope being negative is 100% and the probability of a positive slope is 0%.

7. If the numbers -2 , -4 , and -6 randomly replace the letters a , b , and c in the formula $ax + by = c$, without repetition, what is the probability the slope will be negative? Justify your answer.

If all the values of the replacement numbers are negative, the slopes will all be negative. For any of the possible combinations, solving for y means a sign change for the coefficient of x . This creates unlike signs for the coefficients of x and y ; dividing by the coefficient of y will produce a negative slope.

8. If the numbers 2 , 4 , and -6 randomly replace the letters a , b , and c , in the formula $ax + by = c$, without repetition, what is the probability that the slope will be negative? Justify your answer.

If only one of the numbers has a negative sign, the probability of a negative slope is $1/3$. In order for the slope to be negative, a and b must have the same sign in the original rule. There are 6 possible combinations. Of these, it is only when 2 and 4 replace a and b that the signs are the same. The two possibilities are $2x + 4y = -6$ and $4y + 2x = -6$.

9. If the numbers 2 , -4 , and -6 randomly replace the letters a , b , and c in the formula $ax + by = c$, without repetition, what is the probability that the slope will be negative? Justify your answer.

The values of a and b must have the same sign for the slope to be negative. In question #8, with two positive numbers there was a $2/3$ chance of a positive slope. Now with two numbers negative the chance of a positive slope remains $2/3$. Out of the 6 possible combinations, the paired coefficients with different signs will be: 2 and -4 ; 2 and -6 ; -4 and 2 ; and -6 and 2 . Different signs for the coefficients produce a positive slope thus this situation will produce a positive slope $2/3$ times and the chance of a negative slope is $1/3$. In #8 and #9, 2 out of 6 pairs had the same sign (specifically, -6 and -4 , and -4 and -6); hence, the probability of a negative slope was the same, $1/3$, for both questions.

Math notes

The “big idea” for problems 1 - 5 is that it does not matter what the numbers are in the sample space, the problem is only concerned with the coefficients a and b . If a and b have like signs, then the slope will be negative.

Teaching notes

Have students work problems 1 to 5 and then come back together for summary and discussion. Questions to ask during the discussion:

Problem #1:

- If the students are not able to generate a generalization for the situation, ask them: Would the outcome change if I gave you the numbers 5, 9, and 12?
- What if the numbers were 3, 56, and 103?
- Does it matter what positive numbers are used?

Problem #2: Are there other ways to determine the sample space without listing all the possibilities?

Problem #3: How could you use the information from problem #1 to find the answer to #3?

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2. How many possible combinations of equations are there using the numbers 2, 5, and 7, to replace the letters a , b , and c in the equation $ax + b = c$, if repetition is allowed?
3. If the numbers in question #1 are changed to -2 , -5 , and -7 and then randomly replace the letters a , b , and c without repetition in the equation $ax + b = c$, what is the probability that x will be negative? Justify your answer.
4. Would the probability that x will be negative change if a 4th number were added to the sample space? Justify your answer.
5. If the sample space includes the integers -100 to 100 , excluding 0 , will the probability that x is negative change? Justify your answer.

