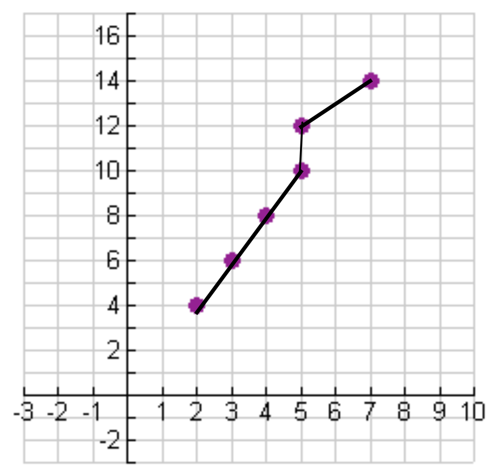
TASK 1.2.3: MATHEMATICAL DEFINITION OF FUNCTIONS

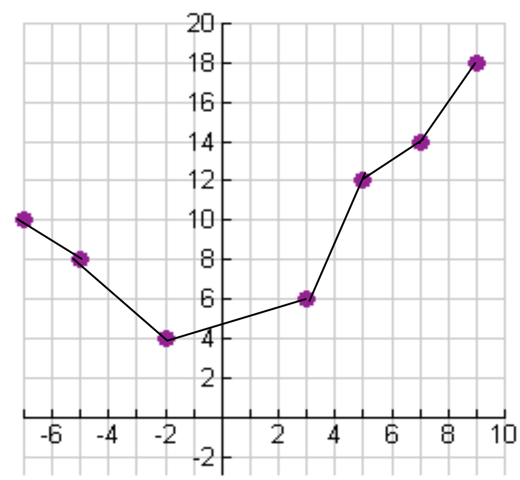
Solutions, Task 1.2.3, Part 1
Look at the following scatter plots, list the coordinates, connect the points from left to right and decide if the data represents a function based on the definition of a function. The definition of a function is for each input value there is only one output value.

A



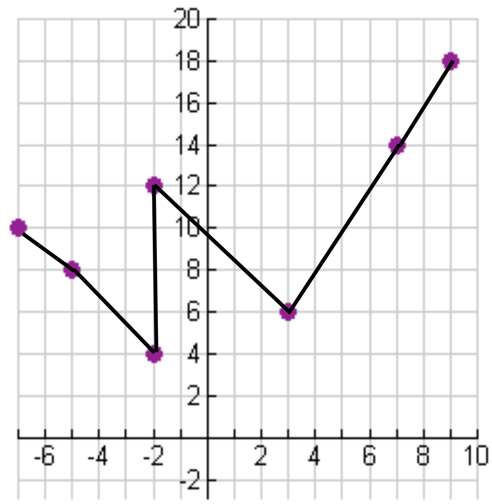
A. The coordinates shown on the graph are (2, 4) (3, 6) (4, 8) (5, 10) (5, 12) and (7, 14). This does not represent a mathematical function based on the definition. One of the input values, 5, resulted in two output values, 10 and 12.

B



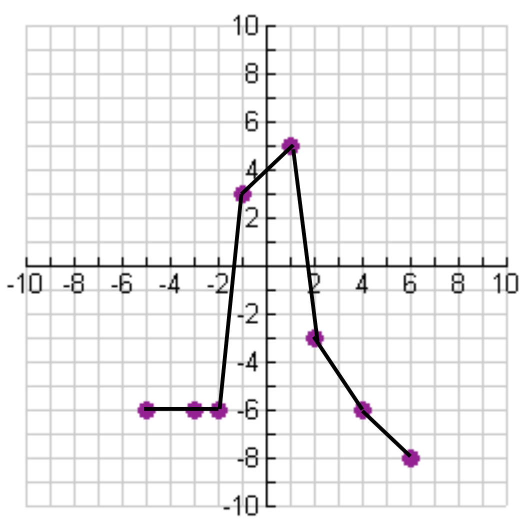
B. The coordinates shown on the graph are (-7, 10) (-5, 8) (-2, 4) (3, 6) (5, 12) (7, 14) and (9, 18). This does represent a function. Each input value has only one output value.

C



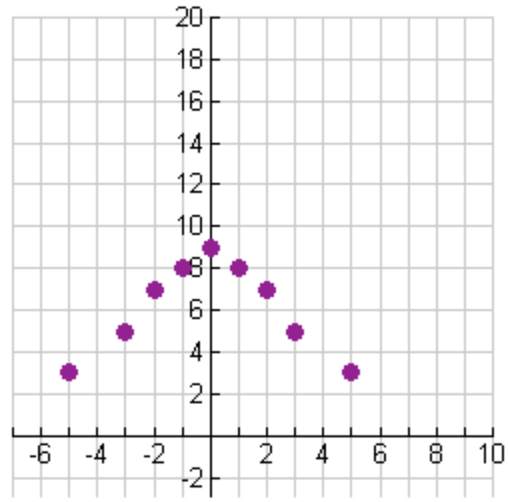
C. The coordinates are (-7, 10) (-5, 8) (-2, 4) (-2, 12) (3, 6) (7, 14) and (9, 18). This does not follow the definition of a function. For an input value of –2, two output values were produced: 4 and 12.

D



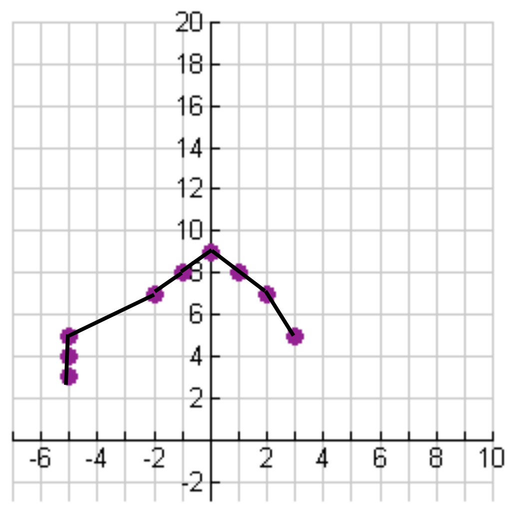
D. The coordinates are (-5, -6) (-3, -6) (-2, -6) (-1, 3) (1, 5) (2, -3) (4, -6) and (6, -8). This does represent a function according to the definition. Each input value (x) results in only one output value.

E



E. The coordinates are (-5, 3) (-3, 5) (-2, 7) (-1, 8) (0, 9) (1, 8) (2, 7) (3, 5) and (5, 3). This follows the definition of a function; each input value (x) results in only one output value. Notice that the in the coordinate points (-3, 5) and (3, 5) the input values produce only one output value.

F



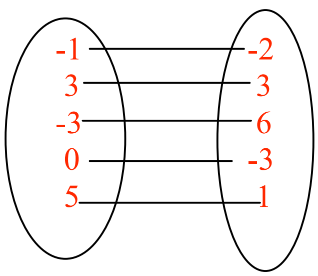
F. The coordinates are (-5, 3) (-5, 4) (-5, 5) (-2, 7) (-1, 8) (0, 9) (1, 8) (2, 7) and (3, 5). This does not match the definition for a function. The coordinates (-5, 3) (-5, 4) and (-5, 5) show an input value of -5 resulting in three output values, not one as the definition requires.

Look at each of the graphs. Make a generalization about when a graph follows the definition of a function? Each of the graphs that did not fit the definition of a function included a vertical line segment. A vertical line segment occurs when at least two output values are matched to the same input value. Therefore, if a vertical line intersects a graph in more than one point, the graph does not represent a function because the points on the vertical line will have the same x-value matched to more than one y-value.

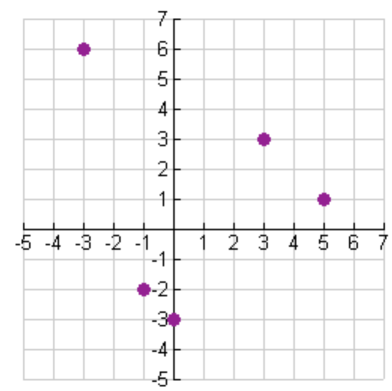
Solutions, Task 1.2.3, Part 2

Use each of the following lists of coordinates; create a mapping, a table of values, and a scatter plot for each group of points. Explain which groups of points follow the definition of a function.

1. (–1, –2) (3, 3) (-3, 6) (0, -3) (5, 1)

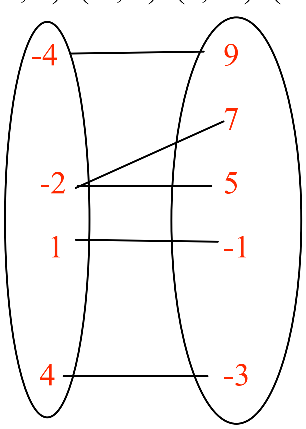


Input	Output
-1	-2
3	3
-3	6
0	-3
5	1

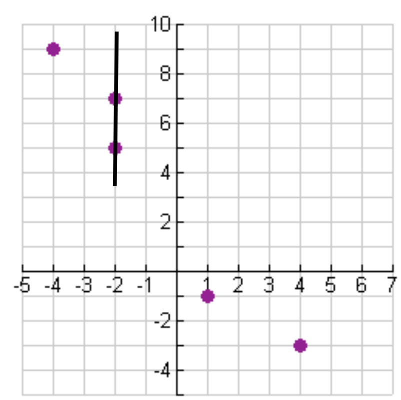


These coordinates follow the definition of a function. Each x-value is mapped to only one y-value.

2. (–4, 9) (-2, 5) (1, -1) (–2, 7) (4, –3)

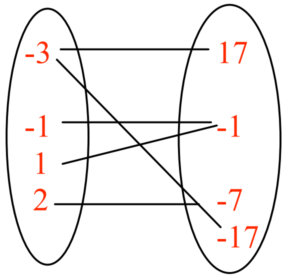


Input	Output
-4	9
-2	5, 7
1	-1
4	-3

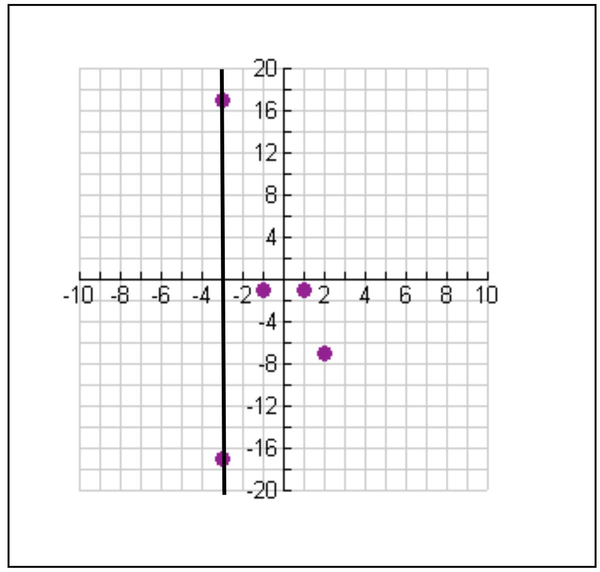


For these points, an input value produces more than one output. This results in a vertical line on the graph and does not match the definition of a function.

3. (-3, 17) (−1, −1) (1, −1) (2, -7) (−3, −17)

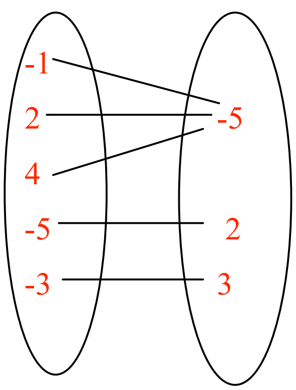


Input	output
-3	17, -17
-1	-1
1	-1
2	-7

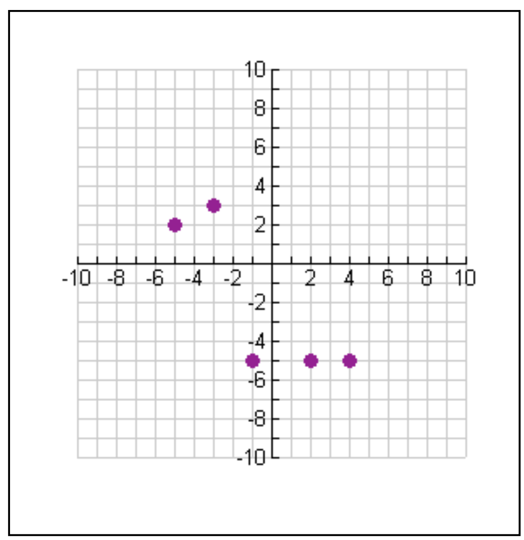


The mapping shows an x-value (input) mapped to more than one output value. This does not match the definition of a function. A vertical line can be seen on the graph.

4. (−1, −5) (2, −5) (4, −5) (−5, 2) (−3, 3)



Input	output
-1	-5
2	-5
4	-5
-5	2
-3	3



The mapping shows that each input is matched to only one output value. Three different input values are mapped to the same output value. This produces a horizontal section on the graph and fits the definition of a function.

Solutions, Task 1.2.3, Part 3

Read each of the following situations and determine whether or not it represents a functional situation. Justify your answer.

1. The student's age in your classroom matched to their shoe size.

This would not represent a functional situation. There could be 2 different 14 year old girls and one could wear a size 7 shoe and the other would wear size 8 shoes. That would map the age of 14 to two shoe sizes. That does not fit the definition of a function. It represents a situation where one input value is mapped to two output values.

2. The numbers on a telephone matched to the letters on the phone.

This does not represent a functional situation. The input values would be 1, 2, 3, etc. and the output values would be the letters. The number 1 on a phone would be mapped to the letters A, B, and C. This is one input value with more than one output value.

3. The letters on the telephone matched to the numbers on the phone.

This does represent a function. Each letter is matched to only one number. Ex. A is mapped to 1, B is mapped to 1, and C is mapped to 1.

4. The months, numbered 1 to 12, matched to the number of days in the month.

This does match the definition for a function. Each number of a month will have only one number for days. January, month number 1 is matched to 31 days and March, month number 3 is matched to 31, No month can have more than one number of days.

5. Any number times 2 plus 5

This statement matches the definition for a function. Every input value will produce only one output value.

6. The cost of renting a car if there is a charge of $0.50 per mile.

This matches the definition for a function. For every mile there can be only one charge.

7. The number of minutes used on a cell phone matched to the students in your class.

This would not fit the definition for a function. 120 minutes on the cell phone may be matched to more than one student.

Math notes: Mathematical definition of function
Students traditionally have difficulty with the definition of a mathematical function. Many times students leave a discussion of functions with only a verbal response of "vertical line test" to questions about functions. Students have a very limited understanding of the definition of a function or the role the vertical line test plays in the definition.

Definitions cannot be discovered or constructed independently by students, but they can be presented contextually, allowing students to apply and develop understanding of the definition.

Class discussion should focus around concrete examples before moving on to mathematical abstractions.

The definition of functions was created to model natural results in life. For each choice presented, a person may choose only one option. Ex. You come to a fork in the road – will you go right or left? You want to put on a shoe, dress shoe, or tennis shoe? You want to buy a vehicle, a car, or a truck? For each time you make a choice, the result is unique to that particular situation.

The definition of a function is based on the same premise – for each input (decision) there can be only one output (choice). Thus $y = 3x + 4$ represents a function, because for each input value, only one output value will be produced. This is evident in the coordinate points produced by this graph, such as (1, 7), (2, 10), and (−8, 20). For every value put into the equation, there will be only one result. The equation $y = 4$ also adheres to the mathematical definition of a function. Each different input value produces only one output value, even if it is the same output value. Sample coordinate points for this function are (1, 4), (3, 4) and (−8, 4). Again each input value results in only one output value.

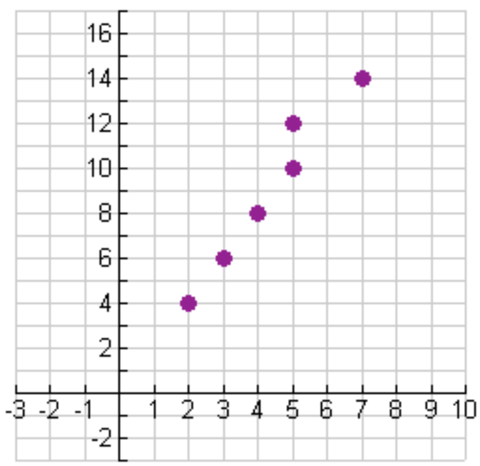
The equation $x = 4$ does not follow the definition for a function. In this case the only input value is 4 and there is not just one matching y value, but rather an infinite number of y values. Looking at some sample coordinate points produced by this equation (4, 1), (4, 2), and (4, −8) show that the same input value gives more than one output value. This is not a function because it does not match the definition.

Task 1.2.3: Mathematical definition of functions

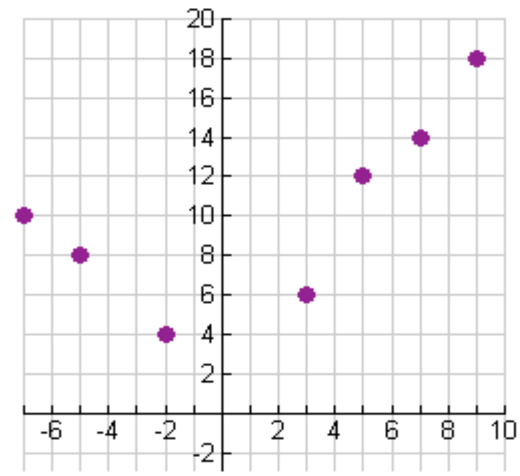
Task 1.2.3, Part 1

Look at the following scatter plots. List the coordinate points, connect the points from left to right, and decide if the data represents a function based on the definition of a function. The definition of a function is for each input value there is only one output value.

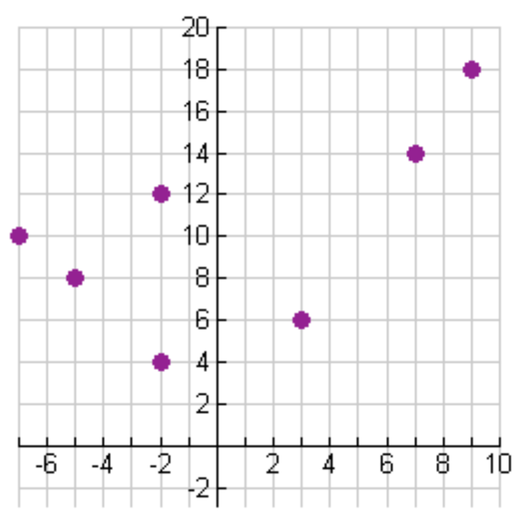
A



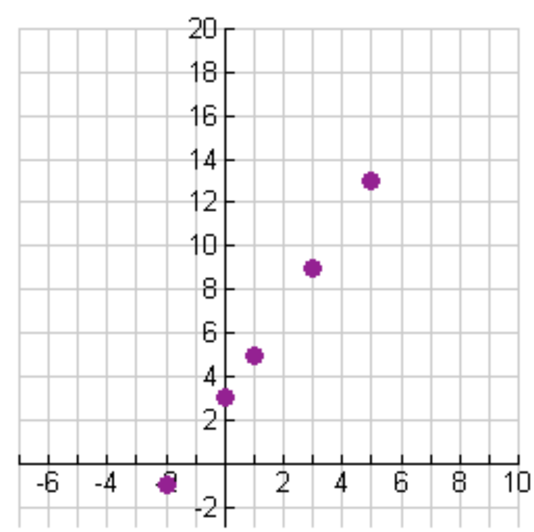
B



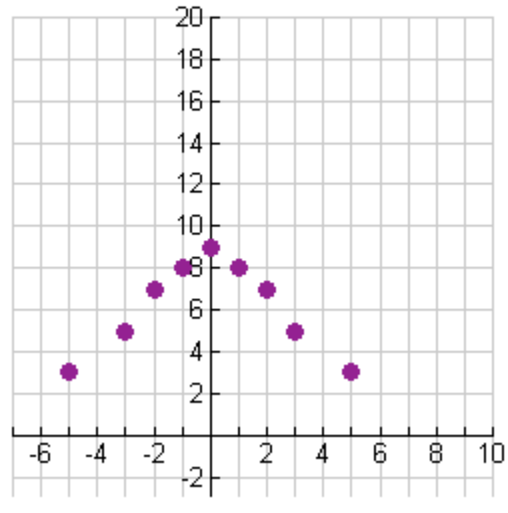
C



D



E



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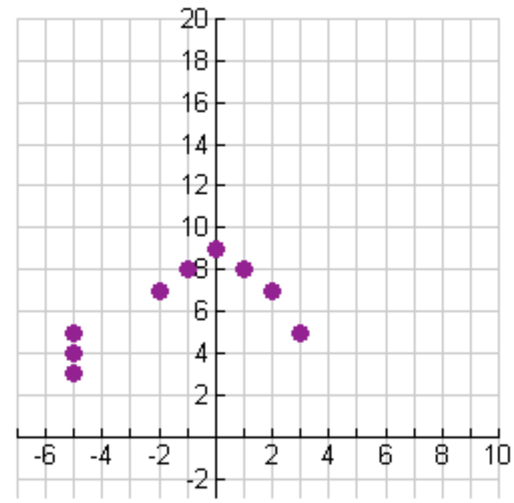

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Task 1.2.3, Part 2

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3. (-3, 17) (−1, −1) (1, −1) (2, -7) (−3, −17)

4. (−1, −5) (2, −5) (4, −5) (−5, 2) (−3, 3)

Task 1.2.3, Part 3

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2. The numbers on a telephone matched to the letters on the phone.
3. The letters on the telephone matched to the numbers on the phone.
4. The months, numbered 1 to 12, matched to the number of days in the month.
5. Any number times 2 plus 5
6. The cost of renting a car if there is a charge of $0.50 per mile.
7. The number of minutes used on a cell phone matched to the students in your class.