
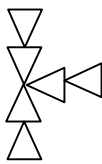
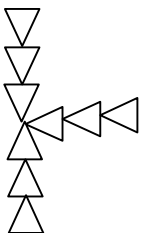


TASK 1.1.1.A: INCREASING PATTERNS WITH DIRECT VARIATION**Solutions**

| Design Number | Visual Form | Written Description | Process Column | Total tiles | Ordered pair |
|--|--|---|-----------------------------|-------------|--------------|
| 1 |  | A figure with 3 legs | $1 + 1 + 1$ or $3(1)$ | 3 | (1, 3) |
| 2 |  | A figure with 3 legs and 2 tiles on each leg | $2 + 2 + 2$ or $3(2)$ | 6 | (2, 6) |
| 3 |  | A figure with 3 legs and 3 tiles on each leg. | $3 + 3 + 3$ or $3(3)$ | 9 | (3, 9) |
| 4 | Students will draw new figure with 4 tiles on each leg | A figure with 3 legs and 4 tiles on each leg. | $4 + 4 + 4$ or $3(4)$ | 12 | (4, 12) |
| 18 | | 3 legs and 18 tiles on each leg | $3(18)$ | 54 | (18, 54) |
| 27 | | 3 legs and 27 tiles on each leg | $3(27)$ | 81 | (27, 81) |
| Write a sentence that describes this pattern For every new design number 3 new tiles are added. Or the pattern has three legs and the design number determines the number of tiles in each leg. | | | | | |
| n | | 3 legs and n tiles on each leg | $3n$ | $3n$ | (n, $3n$) |

(NOTE: an alternate but equivalent way to see this pattern is to begin with a base figure of 3 tiles, then add the design number minus one, $n - 1$, tiles to each of the three legs. See Math notes.)

Answers to second page of pattern #1a

Suppose you are given a box with 114 tiles:

1. Graph your data from the chart on graph paper and/or create a scatter plot on a graphing calculator. What is the domain for this situation? The range? Explain.
The domain is the x-values or input data and in this situation is $\{1, 2, 3, \dots, 38\}$. The range is the y-values or output and in this situation is $\{3, 6, 9, \dots, 114\}$. A good viewing window for your graphing calculator will be larger than your range and domain, allowing all coordinate points to be visible. A good viewing window for this situation could be X_{\min} and X_{\max} of 0 and 40, and Y_{\min} and Y_{\max} of 0 and 120. Range and domain values must be exact but window size may vary.

2. How many total tiles would there be on the 36th design? Explain how you determined this.
There would be a total of 108 tiles in the 36th design. The design number is an input value in this situation. Choosing any of the equivalent expressions and inputting 36 will result in an output of 108.

Example: $3(36)=108$ or $3+3(36-1)=108$

3. Which term would have 36 total tiles? How do you know?
There would be 36 total tiles on the 12th design. The number of tiles is an output value and the design number is an input value. You can use any of the equivalent expressions and solve for the input value.

Example: $3n=36$ or $3+3(36-n)=36$ both result in a design number, n , of 12.

4. What does the ordered pair (5, 15) mean in this situation?
Five is the design number and 15 is the number of tiles it would take for the 5th design.

5. Would the point (16, 46) fit this pattern? Explain why or why not.
No, (16, 46) does not fit this pattern. If it were the 16th design, there would be 16 tiles on each of the 3 legs. That makes a total of 48 tiles.

6. Use the picture to explain the pattern.
As each new design is created, the previous design is modeled and one new tile is added to each leg.

7. Explain how the table reflects the pattern.
In the table, the total number of tiles grows by three for each new design number.

8. Explain how the pattern can be seen in the scatter plot.
On the scatter plot, as each design number moves one unit to the right the total number of tiles moves up 3.

Math notes for pattern #1a

Students may also see the pattern as beginning with a base figure of 3 tiles, then adding the design number minus one, $n - 1$, tiles to each of the three legs. This gives a general rule of $t = 3 + 3(n - 1)$. If students see this pattern, that will change their written description and their process column entries. The table values stay the same; the graph is the same. Using the distributive property and combining terms shows that all of the rules are algebraically equivalent expressions, that is $n + n + n = 3n = 3 + 3(n - 1)$. However, the algebraic rule that students find needs to reflect the way they “see” the pattern in their concrete or pictorial model.


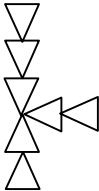
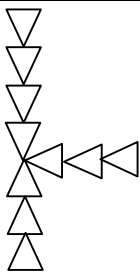
Teaching notes

In this lesson, we **are not** trying to teach about sequences nor do we want to have any discussion on the **$mx + b$** form. We in fact do not care whether or not the student uses these letter designations. We want to have them start **recognizing the $(0,b)$** pair. We also want them to see that many patterns are in the form:

**Constant * variable or
constant * variable + constant**

Make sure students relate the concrete model change to the graphical representation and the tabular representation. (Do not introduce finite differences at this point.)

TASK 1.1.1.B: INCREASING PATTERNS WITHOUT DIRECT VARIATION**Solutions**

| Pattern Number | Visual Form | Written Description | Process Column | Total tiles | Ordered pair |
|--|--|--|----------------|-------------|--------------|
| 1 |  | Top leg with 2 tiles and 2 legs with one tile | $2 + 2(1)$ | 4 | (1, 4) |
| 2 |  | Top leg with 3 tiles and 2 legs with 2 tiles | $3 + 2(2)$ | 7 | (2, 7) |
| 3 |  | Top leg with 4 tiles and 2 legs with 3 tiles | $4 + 2(3)$ | 10 | (3, 10) |
| 4 | | Top leg with 5 tiles and 2 legs with 4 tiles | $5 + 2(4)$ | 13 | (4, 13) |
| 18 | | Top leg with 19 tiles and 2 legs with 18 tiles | $19 + 2(18)$ | 55 | (18, 55) |
| 27 | | Top leg with 28 tiles and 2 legs with 27 tiles | $28 + 2(27)$ | 82 | (27, 82) |
| Write a sentence that describes what is happening to the pattern. In this pattern 1 leg has one more than the design number and 2 legs have the same number of tiles as the design number. | | | | | |
| n | | | $(n+1) + 2n$ | $3n+1$ | (n, $3n+1$) |

Other, but equivalent, ways to describe this pattern:

Start with a base pattern of 4, then add the design number minus one, $n - 1$, tiles to each leg.
 $4 + 3(n - 1)$.

Algebra I: Strand 1. Foundations of Functions; Topic 1. Identifying Patterns; Task 1.1.1a & b

Start with the one top triangle in each figure, then add the design number of tiles, n , to build each of the three legs.

Answers to second page of pattern #1b

Suppose you are given a box with 115 tiles:

1. Graph your data on graph paper and/or create a scatter plot on a graphing calculator. What is a reasonable domain for this situation? A reasonable range? Explain.

For this situation, a reasonable domain for this data is the integers from 0 to 38. A reasonable range is the integers 1 to 115. Refer to math notes for clarification.

2. How does this pattern and expression differ from the first pattern?

There is one extra tile on the first pattern; this means that every pattern will have 1 more tile than the first pattern.

3. How many total tiles would there be on the 37th term? Explain how you know.

There would be a total of 112 tiles in the 37th pattern. The design number in this situation is an input value. There are several ways to explain this result. Using any of the equivalent expressions developed for this pattern input 37 to derive the output value of 112.

Example: $(n+1)+2n=(37+1)+2(37)=112$

Use the table feature on the graphing calculator

4. Would there be a pattern that would have 37 total tiles? Justify your answer.

The 12th pattern would have 37 tiles. The total number of tiles is an output value. Using any of the equivalent expressions and solving for 37 will determine the corresponding input value. Type the rule into $Y=$ on the graphing calculator and use the trace feature to find the corresponding x -value when $y=37$. Use the table on the graphing calculator to find the corresponding x -value when $y=37$.

5. What does the coordinate point (15, 46) mean in this pattern situation?

Fifteen is the design number and 46 is the number of tiles it would take for the 15th pattern.

6. Using the picture explain how the pattern is changing.

The pattern starts with 1 tile and adds 3 tiles for each design number.

7. Explain how the table reflects the change in the pattern.


In the table the total number of tiles grows by three for each new pattern.

8. Explain how the change in the pattern can be seen in the scatter plot.


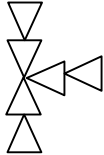
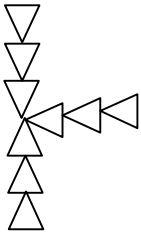
On the scatter plot – The starting point is 1 and each design number moves one unit to the right while the total number of tiles moves up 3.

Math notes for pattern #1b

There are many different (but equivalent) rules that could be generated from this pattern. It is essential that students connect their rule with what they see in the concrete or pictorial models. For example, if the 4-tile pattern #1 is seen as the base structure, then each design consists of the original 4 tiles plus additional tiles on each leg that number one less than the design number, $4 + 3(n - 1)$. Teachers need to be able to recognize more than one rule for the pattern. The need for distributive, commutative, and associative properties arises from the different, yet equivalent, rules; i.e. $(n + 1) + 2n = 3n + 1$ and $4 + 3(n - 1) = 4 + 3n - 3 = 3n + 1$. The rules, however, indicate very different ways of building the pattern geometrically. We want students to recognize that data generated from each of the different rules gives the same graph and table values.



TASK 1.1.1.A: INCREASING PATTERNS WITH DIRECT VARIATION


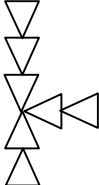
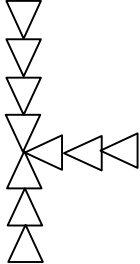
| Design Number | Visual Form | Written Description | Process Column | Total tiles | Ordered pair |
|--|---|---------------------|----------------|-------------|--------------|
| 1 |  | | | | |
| 2 |  | | | | |
| 3 |  | | | | |
| 4 | | | | | |
| 18 | | | | | |
| 27 | | | | | |
| Write a sentence that describes this pattern | | | | | |
| n | | | | | |

Identifying Patterns #1a

Suppose you are given a box with 114 tiles:

1. Graph your data from the table on graph paper and/or create a scatter plot on a graphing calculator. What is the domain for this situation? The range? Explain.
2. How many total tiles would there be on the 36th design? Explain how you determined this.
3. Which design would have 36 total tiles? How do you know?
4. What does the ordered pair (5, 15) mean in this situation?
5. Would the point (16, 46) fit this pattern? Explain why or why not.
6. Use your picture to explain the pattern.
7. Explain how the table reflects the pattern.
8. Explain how the pattern can be seen in the scatter plot.

TASK 1.1.1.B: INCREASING PATTERNS WITHOUT DIRECT VARIATION

| Design Number | Visual Form | Written Description | Process Column | Total tiles | Ordered pair |
|--|--|---------------------|----------------|-------------|--------------|
| 1 |  | | | | |
| 2 |  | | | | |
| 3 |  | | | | |
| 4 | | | | | |
| 18 | | | | | |
| 27 | | | | | |
| Write a sentence that describes this pattern | | | | | |
| n | | | | | |

Identifying Patterns #1b

Suppose you are given a box with 115 tiles:

1. Graph your data from the table on graph paper and/or create a scatter plot on a graphing calculator. What is the domain for this situation? The range? Explain.
2. How does this pattern and expression differ from the first pattern?
3. How many total tiles would there be on the 37th design? Explain how you know.
4. Would there be a design that would have 37 total tiles? Justify your answer.
5. What does the coordinate point (15, 46) mean in this pattern situation?
6. Use your picture to explain the pattern.
7. Explain how the table reflects the pattern.
8. Explain how the pattern can be seen in the scatter plot.