IV. COURSE CONTENT

A. INTRODUCTION

Including introductory materials for Algebra I *and* Algebra II *course content.*

To ensure that all teacher participants—and ultimately all Texas teachers who complete the Algebra I and/or Algebra II course—gain the necessary knowledge and skills to help students meet state content and student performance standards, we have developed a teacher training course that focuses on *function*—a core idea of Algebra I as articulated in the Texas Essential Knowledge and Skills.

The *Ensuring Teacher Quality: Algebra I* content focuses on foundations for functions (developing mathematical models, using patterns to identify relationships, and interpreting graphs), linear functions (including an in-depth look at linear functions, interpreting relationships between data sets, and linear equations and inequalities), and nonlinear functions (quadratic functions and equations, exponential functions and equations, and interpreting relationships between data sets).

Because data show that problem solving is still a weak area across the mathematics continuum, problem solving is developed as a vertical strand that is a connecting piece throughout the Algebra I and Algebra II mathematics modules. In this module, we will focus on problem solving in Algebra I, including solving nonroutine problems, why and how to teach problem solving, teaching for mathematical proficiency, improving teacher questioning strategies, and determining what makes a good problem. The developers for the *Ensuring Teacher Quality: Algebra I* project have created at least 15 days of content to ensure that materials will be at least a 3-hour credit course. (A course syllabus is at II. 2., within the Overview for the Facilitator.)

Regarding common *content development and delivery strategies*, we have learned from our experience that for teacher training materials to be effective, there must be active, up-front involvement of all stakeholders (including teachers, professional developers, administrators, and university faculty); intense effort to ensure quality, appropriateness, and consistency; and adequate time for teachers to collaborate.

This section contains resources to help trainers and teachers think about:

1. Problem solving across the curriculum
2. Support for struggling students
3. Technology use throughout mathematics

A list of references cited appears at the end of each part.

1. Problem solving across the curriculum

What is problem solving?

As defined by the National Council of Teachers of Mathematics (p. 52, 2000), "problem solving means engaging in a task for which the solution method is not known in advance." There is a difference between an exercise (whether a word problem or not) where solution methods have been demonstrated, and a problem, where no solution method has been demonstrated for students. "Problem solving is both a way of thinking and a way of doing mathematics (p. 5, Hyde and Hyde, 1991)."

Why is problem solving important?

In the Texas Essential Knowledge and Skills (TEKS), both teachers and students are expected to use problem solving as an underlying process in all content areas. This expectation is reflected in Objective 10 of the Texas Assessment of Knowledge and Skills (TAKS): "The student will demonstrate an understanding of the mathematical processes and tools used in problem solving." This objective has historically had lower statewide results than other objectives in mathematics across all grade levels. Therefore it continues to be a high priority for all mathematics teachers.

- "By learning problem solving in mathematics, students should acquire ways of thinking, habits of persistence and curiosity, and confidence in unfamiliar situations that will serve them well outside the mathematics classroom. In everyday life and in the workplace, being a good problem solver can lead to great advantages" (p. 52, NCTM, 2000).

- "We want students to be engaged and motivated, to become self-reliant, to feel competent and capable of generating and investigating questions, collecting and analyzing information, and solving problems in school and in life" (p. 10, Hyde and Hyde, 1991).

What does problem solving encompass?

According to Schoen (p. xi, NCTM, 2003) problem solving requires access to tasks that include the mathematics that is to be learned; are built on what students already know and can do; and are presented in an engaging and accessible way. The teacher must support student learning through problem solving; provide access to appropriate technology; and ensure that the understanding that comes from learning mathematics through problem solving is accessible to all students.

- "Problem solving is an integral part of all mathematics learning, so it should not be an isolated part of the mathematics program. Problem solving in mathematics should involve all five content areas described in the NCTM *Principles and Standards for School Mathematics* (2000). The contexts of the problems can vary from familiar experiences involving students' lives or the school day to applications involving the sciences or the world of work. Good problems will integrate multiple topics and will involve significant mathematics" (p. 52, NCTM, 2000).

- "Is it really possible to go through the entire curriculum creating problem solving activities for every concept? Yes, and it is not as mammoth a task as one might imagine. First, the number of fundamental concepts to be taught in a year is large but not enormous. Second, some concepts require less experiential problem-solving activities than others, for two reasons: (1) students may already have good conceptions . . . to build upon, and (2) many concepts can be related to one another" (p. 10, Hyde and Hyde, 1991).

How are good problems designed to elicit problem solving?

As Hyde and Hyde (pp. 15–16, 1991) explain, a vital step in using problem solving to teach mathematics is to design good problems that bring together the world of mathematics and the world of the students. Some criteria of good problems are the following:

- "There is no obvious way to work on the problem.
- "The problem is set in a meaningful context for the students.
- "The problem provokes students' interest in pursuing it.
- "Working on the problem uses mathematical thinking and knowledge that is developmentally appropriate.
- "Discussion of solutions allows the teachers to build on the problem to explore mathematical ideas."

Changing existing exercises or problems into rich problem-solving activities takes practice. The following table lists components of a rich mathematics problem.

What is a rich mathematics problem?	**What is not rich?**
Leads to important mathematical ideas	An answer that leads nowhere
Challenging, takes time	Recall/One answer
Solvable in many ways	Only one approach
Stimulates other questions/discussions	Skill rather than concept/analysis
Makes connections between different mathematical fields	No connections
Leads to new interesting problems	Does not lead to new problems

How can rich problems be created from ordinary textbook problems?

Following are suggestions identified by Epperson and Kribs-Zaleta (2004) to facilitate the process of changing an ordinary textbook problem into a rich problem; however, the mathematical idea is essential in any rich task.

- Ask for prediction and comparison
- Ask for examples or counterexamples
- Ask for generalization, e.g., families of functions
- Invert the problem (ask the question in a nontraditional way)
- Ask students to reconsider a problem while changing a few conditions
- Ask questions that require qualitative reasoning, e.g., explain relationships
- Ask for multiple representations

References for Problem Solving

Epperson, J. & Kribs-Zaleta, C. (March 5–6, 2004). “Creating rich mathematics problems from standard textbook problems.” Presentation at Conference: *Supporting and Strengthening Standards-based Mathematics Teacher Preparation Faculty Retreats*.

Hyde, A. A. & Hyde, P. R. (1991). *Mathwise: Teaching Mathematical Thinking and Problem Solving*. Portsmouth, NH: Heinemann, a division of Reed Elsevier, Inc.

National Council of Teachers of Mathematics. (2000). *Principles and Standards for School Mathematics*. Reston, VA: NCTM.

National Council of Teachers of Mathematics. (2003). *Teaching Mathematics through Problem Solving: Grades 6–12*. Harold L. Schoen (Vol. Ed.), Randall I. Charles, (Series Ed.). Reston, VA: NCTM.

2. Support for struggling students

Who are struggling students?

Every student is potentially a struggling student. A student in a mathematics classroom may struggle for a variety of reasons. Groups of struggling students may include students with:

- Learning disabilities
- Challenging home environments (e.g. homelessness, mobility)
- Success in other academic areas but a weakness in mathematics
- Difficulties in just one area of mathematics (Kroesbergen, 2003)
- Lack of motivation or “poor fit” with traditional instructional methods (Kroesbergen, 2003)

Why is the struggling student an important topic?

For both new and experienced teachers, there is an ongoing need for lessons, strategies, and assessments that support learning for struggling students. While most teacher preparation programs briefly address the needs of special populations, there is rarely a focused support for these groups in the area of mathematics. This topic is also important for the following reasons:

- All students in Texas are required to take Algebra I to graduate from high school.
- Students going to college or graduating with the Recommended High School Program need Algebra II.
- The strategies that work for struggling students are good for all students (Kilpatrick, et al., 2001).
- Teacher–student interactions make a big difference in the success of struggling students (Empson, 2003).
- Preservice teachers need instruction to best use the methods that work for struggling students (Learning First Alliance, 1998).

What works for struggling students?

Mathematics education research continues to investigate what works for struggling students. While much of this research is at the elementary school level, some extrapolation is reasonable. The following suggestions should be considered as teachers plan:

- Formative assessment—Assess student learning throughout a course. Teachers should use data to modify instruction, and students should monitor their progress to seek improvement (Boston, 2002).
- Peer tutoring—Support tutoring with the struggling student both receiving and providing the tutoring (Allsopp, 1997; Cheung, 1999).
- Technology—Classrooms should include calculators, computer programs, videos, etc. (NCTM, 2000).
- Hands-on activities and problem solving opportunities—Give struggling students opportunities to engage in meaningful mathematical activities (Bottge, et al., 2001).
- Build on the student's prior knowledge (Kilpatrick, et al, 2001).

References for Struggling Students

Allsopp, D. (1997). Using classwide peer tutoring to teach beginning Algebra problem solving skills in heterogeneous classrooms. *Remedial and Special Education*, *18*(6), 367–380.

Boston, C. (2002). The concept of formative assessment. *Practical Assessment, Research and Evaluation: A peer-reviewed electronic journal*, *8*(9), available through pareonline.net/.

Bottge, B., Heinrichs, M., Shih-Yi, C. & Serlin, R. (2001). Anchoring adolescents' understanding of math concepts in rich problem-solving environments. *Remedial and Special Education, 22*(2), 102–112.

Cheung, C. & Winter, S. (1999). Classwide peer tutoring with or without reinforcement: effects on academic responding, content coverage, achievement, intrinsic interest and reported project experiences. *Educational Psychology*, *19*(2), 191–216.

Empson, S. B. (2003). Low performing students and teaching fractions for understanding: An interactional analysis. *Journal for Research in Mathematics Education, 34*(4), 305–343.

Kilpatrick, J., Swafford, J. & Findell, B. (Eds.) (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academy Press.

Kroesbergen, E. & Van Luit, J. (2003). Mathematics interventions for students with special educational needs. *Remedial and Special Education, 24*(2), 97–115.

Learning First Alliance. (1998). *Every child mathematically proficient: An action plan.* Washington, DC: Learning First Alliance.

National Council of Teachers of Mathematics. (2000). *Principles and Standards for School Mathematics*. Reston, VA: NCTM.

3. Technology use throughout mathematics

What technology?

Advances in technology have affected the way we teach mathematics. The types of technology we most often refer to are:

- Graphing calculators
- Motion detectors
- Computer software, such as
  - Spreadsheets (Excel)
  - Geometer's Sketchpad or Cabri Geometry
- Online curriculum resources

Why is technology an important topic?

"The existence, versatility, and power of technology make it possible and necessary to reexamine what mathematics students should learn as well as how they can best learn it" (NCTM, 2000).

The high school mathematics Texas Essential Knowledge and Skills and Texas Assessment of Knowledge and Skills require the use of technology.

Technology offers teachers options for adapting instruction to special student needs (NCTM, 2000).

While much research has been conducted on the use of technology in mathematics, the following excerpt reflects some common findings:

> In three large-scale algebra studies, where technology was used in concert with curricular materials, there were gains in advanced understanding of variables, improved ability to solve algebraic problems set in realistic contexts, improved understanding of graphical representations and applications, and no significant differences in procedural skills (p. 31, Burrill, 2002).

How is technology used?

The technologies we have available are used by students to enhance learning by:

- furnishing visual images of mathematical ideas;
- examining additional examples or representational forms;
- making and exploring conjectures easily;
- exploring the effects of the various dynamic transformations that technology allows;
- facilitating the organization and analysis of data; and
- computing efficiently and accurately. (NCTM, 2000)

"When technological tools are available, students can focus on decision-making, reflection, reasoning, and problem solving" (p. 25, NCTM, 2000).

The technologies we have available are used by teachers to enhance teaching by:

- extending the range of problems accessible to students;
- selecting or creating mathematical tasks that take advantage of what technology can do efficiently and well—graphing, visualizing, and computing;
- using simulations to give students experience with problem situations that are difficult to create without technology; and
- using data and resources from the Internet and the World Wide Web to design student tasks (NCTM, 2000).

"Initially, the teacher must decide if, when, and how technology will be used. …As students work with technology, they may show ways of thinking about mathematics that are otherwise often difficult to observe. Thus, technology aids in assessment, allowing teachers to examine the processes used by students in their mathematical investigations as well as the results, thus enriching the information available for teachers to use in making instructional decisions" (p. 25, NCTM, 2000).

Research has found that "because students using handheld graphing calculator technology learn to solve problems using multiple methods, teachers should be prepared to help students examine those methods to see when they generalize or what assumptions or limitations might be inherent in a particular method" (p. 37, Burrill, 2002).

The technologies we have influence what mathematics is taught, by:

- allowing students to reason about more-general issues, such as parameter changes;
- allowing students to model and solve complex problems that were heretofore inaccessible to them;
- blurring some of the artificial separations among topics in algebra, geometry, and data analysis by allowing students to use ideas from one area of mathematics to better understand another area of mathematics; and
- changing the boundaries of the mathematical landscape, such as with discrete mathematics (p. 26, NCTM, 2000).

References on Technology

Burrill, G. et al. (2002). *Handheld Graphing Technology at the Secondary Level: Research Findings and Implications for Classroom Practice.* Dallas, TX: Texas Instruments.

National Council of Teachers of Mathematics. 2000. *Principles and Standards for School Mathematics*. Reston, VA: NCTM.