

## In-Class Activity 1.C Pendulum Motion

- 1) Observe a model (or a video) of a moving pendulum.  
 Write three things that you notice about its motion.



Credit: iStockphoto/Vajirawich Wongpuvarak

### Objectives for the activity

You will understand that:

- The motion of a swinging pendulum can be modeled by sinusoidal functions.
- Sinusoidal functions are periodic, and the period is an important characteristic of the function.

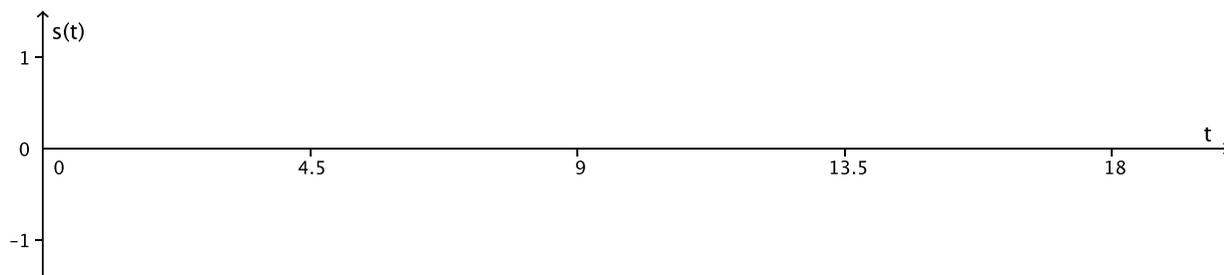
You will be able to:

- Identify the period of a sinusoidal function from its graph.
- Given the graph of a sinusoidal position function, sketch the graph of the corresponding velocity function.

The pendulum exhibit at the Museum of Science and Industry in Chicago has a moving pendulum that is nearly *frictionless*; the weight is very heavy and the string is very long and light, so the pendulum appears to swing back and forth the same distance each time. It takes about 9 seconds for this pendulum to travel back and forth.

- 2) We can represent the position of the pendulum as a function of the time  $t$  in seconds. We will call this function  $s$ . For convenience, let  $s(t) = 1$  when the pendulum is at one extreme (all the way to the right) and let  $s(t) = -1$  when the pendulum is at its left extreme. Suppose that the pendulum starts at its right extreme ( $s(0) = 1$ ).

Part A: On the axes shown, plot the points of this function in the interval  $0 \leq t \leq 18$  that correspond to the pendulum being in the middle and at both extremes of its motion.



Part B: Add a curve to the graph to show the position of the pendulum at all points in the interval  $0 \leq t \leq 18$ . Your curve should account for the following properties of pendulum motion:

- The pendulum slows down gradually before it reverses direction.
- Its movement to the right is the same as its movement to the left, only reversed.

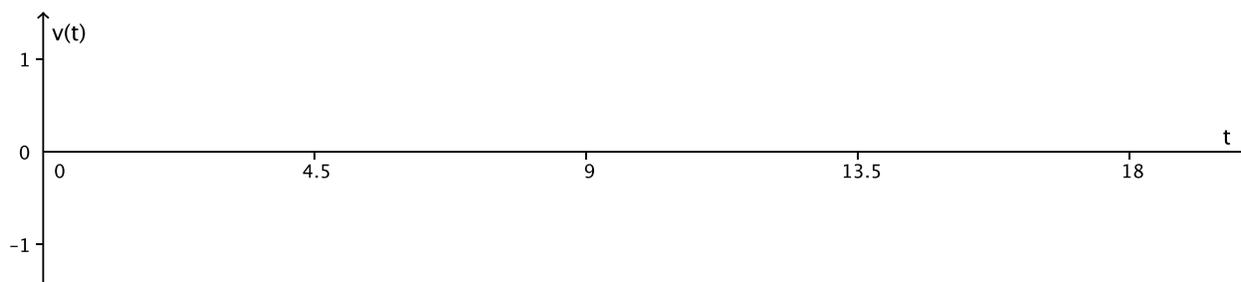
Part C: What characteristics of your graph correspond to the two properties of pendulum motion specified in Part B?

- 3) Consider a function  $v$  that gives the velocity of the pendulum at time  $t$ . Again, for convenience, instead of using standard units for the output, let  $v(t)$  take the value 1 when the pendulum is moving its fastest to the right, and let  $v(t) = -1$  be the fastest leftward velocity.

Part A: At what values of  $t$  in the interval  $0 \leq t \leq 18$  is  $v(t) = 0$ ?

Part B: At what values of  $t$  in the interval  $0 \leq t \leq 18$  is  $v(t) = 1$ ? At what values of  $t$  is  $v(t) = -1$ ?

Part C: On the axes shown, sketch a graph of the function  $v$  on the interval  $0 \leq t \leq 18$ .



Part D: Compare your graphs from Question 2, Part B, and Question 3, Part C. How are they similar? How are they different?

Part E: What is happening to the pendulum when the velocity is zero?

- 4) A function that repeats its values over regular intervals is called a **periodic function**. The **period** of a periodic function  $f$  is the smallest positive number  $P$  such that  $f(x) = f(x + P)$  for all  $x$ . The functions describing the position and velocity of the pendulum are periodic. What are the periods of the functions  $s$  and  $v$  of Questions 2 and 3, respectively?

- 5) The period  $P$  of a pendulum in seconds depends on its length  $L$  in meters according to the formula  $P \approx 2\sqrt{L}$ . Use this formula to approximate the length of the pendulum in the Museum of Science and Industry in Chicago.
- 6) The Panthéon in Paris houses a pendulum having a length of 67 meters. How long would it take for this pendulum to swing back and forth one time?
- 7) The graphs that you drew in this activity and in In-class Activity 1.A have the same basic shape. Functions whose graphs have this shape are called **sinusoidal**. List three properties that you notice about the graphs of sinusoidal functions.

