In-Class Activity 1.C Pendulum Motion

1) Observe a model (or a video) of a moving pendulum. Write three things that you notice about its motion.

Credit: iStockphoto/Vajirawich Wongpuvarak

Objectives for the activity

You will understand that:

- ☐ The motion of a swinging pendulum can be modeled by sinusoidal functions.
- ☐ Sinusoidal functions are periodic, and the period is an important characteristic of the function.

You will be able to:

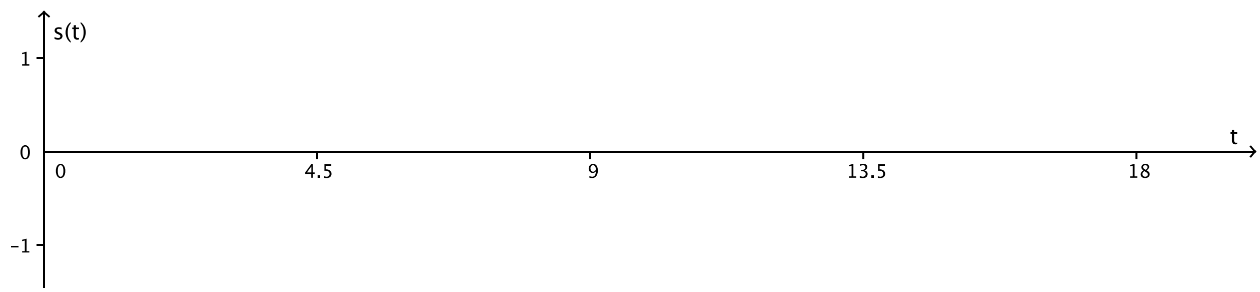
- ☐ Identify the period of a sinusoidal function from its graph.
- ☐ Given the graph of a sinusoidal position function, sketch the graph of the corresponding velocity function.

The pendulum exhibit at the Museum of Science and Industry in Chicago has a moving pendulum that is nearly *frictionless*; the weight is very heavy and the string is very long and light, so the pendulum appears to swing back and forth the same distance each time. It takes about 9 seconds for this pendulum to travel back and forth.

[Continued on the next page.]

2) We can represent the position of the pendulum as a function of the time t in seconds. We will call this function s. For convenience, let $s(t) = 1$ when the pendulum is at one extreme (all the way to the right) and let $s(t) = -1$ when the pendulum is at its left extreme. Suppose that the pendulum starts at its right extreme ($s(0) = 1$).

Part A: On the axes shown, plot the points of this function in the interval $0 \leq t \leq 18$ that correspond to the pendulum being in the middle and at both extremes of its motion.



Part B: Add a curve to the graph to show the position of the pendulum at all points in the interval $0 \leq t \leq 18$. Your curve should account for the following properties of pendulum motion:

- The pendulum slows down gradually before it reverses direction.
- Its movement to the right is the same as its movement to the left, only reversed.

Part C: What characteristics of your graph correspond to the two properties of pendulum motion specified in Part B?

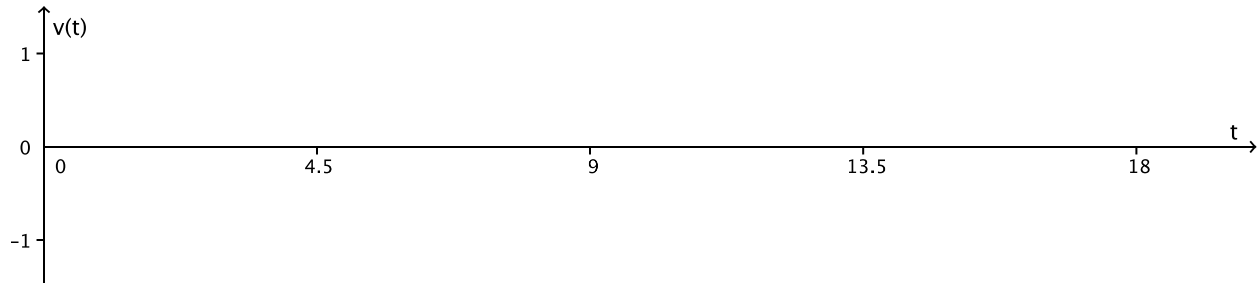
3) Consider a function v that gives the velocity of the pendulum at time t. Again, for convenience, instead of using standard units for the output, let $v(t)$ take the value 1 when the pendulum is moving its fastest to the right, and let $v(t) = -1$ be the fastest leftward velocity.

Part A: At what values of t in the interval $0 \leq t \leq 18$ is $v(t) = 0$?

Part B: At what values of t in the interval $0 \leq t \leq 18$ is $v(t) = 1$? At what values of t is $v(t) = -1$?

[Continued on the next page.]

Part C: On the axes shown, sketch a graph of the function v on the interval $0 \leq t \leq 18$.



Part D: Compare your graphs from Question 2, Part B, and Question 3, Part C. How are they similar? How are they different?

Part E: What is happening to the pendulum when the velocity is zero?

4) A function that repeats its values over regular intervals is called a **periodic function**. The **period** of a periodic function f is the smallest positive number P such that $f(x) = f(x + P)$ for all x. The functions describing the position and velocity of the pendulum are periodic. What are the periods of the functions s and v of Questions 2 and 3, respectively?

5) The period P of a pendulum in seconds depends on its length L in meters according to the formula $P \approx 2\sqrt{L}$. Use this formula to approximate the length of the pendulum in the Museum of Science and Industry in Chicago.

6) The Panthéon in Paris houses a pendulum having a length of 67 meters. How long would it take for this pendulum to swing back and forth one time?

7) The graphs that you drew in this activity and in In-class Activity 1.A have the same basic shape. Functions whose graphs have this shape are called **sinusoidal**. List three properties that you notice about the graphs of sinusoidal functions.