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Launch Years initiative

Modern Algebra II Course Framework

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Launch Years
an initiative of

The University of Texas at Austin
Charles A. Dana Center

CCRC Community College Research Center
Teachers College, Columbia University

Association of Public & Land-Grant Universities

ESG Education Strategy Group

About the Dana Center

The Charles A. Dana Center develops and scales mathematics and science education innovations to support educators, administrators, and policymakers in creating seamless transitions throughout the K–16 system for all students, especially those who have historically been underserved. We focus in particular on strategies for improving student engagement, motivation, persistence, and achievement.

The Center was founded in 1991 at The University of Texas at Austin. Our staff members have expertise in leadership, literacy, research, program evaluation, mathematics and science education, policy and systemic reform, and services to high-need populations.

About Launch Years

Launch Years is an initiative led by the Charles A. Dana Center at The University of Texas at Austin—in collaboration with Community College Research Center, Education Strategy Group, and the Association of Public and Land-grant Universities—focused on addressing systemic barriers that prevent students from succeeding in mathematics and progressing to postsecondary and career success. Leveraging work within states, the initiative seeks to modernize math in high school through relevant and rigorous math courses as well as through policies and practices leading to more equitable outcomes for all students. Learn more at: **utdanacenter.org/launch-years**.

For queries, please contact us at dana-k12@austin.utexas.edu.

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Over the last 20 years, mathematics has become increasingly important to a growing number of fields of study and their related professions. In 1998, the National Science Foundation released "International Assessment of the U.S. Mathematical Sciences" (Odom Report), which listed 11 fields of study that interface with mathematics, including physics, chemistry, economics, and manufacturing. The National Research Council's 2013 report, "The Mathematical Sciences in 2025," expanded this number to 21 and predicted continued growth. The new fields of study added to the list were notable—entertainment, social networks, ecology, computer science, information processing, marketing, and defense.

Unfortunately, the current system of mathematics education fails to meet the needs of many of our students. **This is unacceptable.** It is reprehensible that so many students' opportunities to succeed are limited by their race or economic class. We have a moral and a professional obligation to create the conditions necessary for every student to succeed.

The Launch Years initiative intends to create those conditions through two overarching aims.

The first is to improve learning opportunities for each student during the last two years of high school and into the transition to their postsecondary education and other future endeavors.

The second is to dismantle institutional and systemic barriers that block equitable access and opportunities to succeed in mathematics, especially for students who are Black, Latinx, or Native American, or who come from low-income communities.

The Launch Years vision is to build, scale, and sustain policies, practices, and structures that ensure that each student has equal access to, and successfully engages in:

- **mathematics courses** with rigorous, relevant, engaging, high-quality, and inclusive instruction that is responsive to the needs of individual students and that is informed by multiple measures of achievement that are economically and culturally inclusive;
- **mathematics pathways** that are well articulated from high school to and through postsecondary education and careers, that are personally and socially relevant, and that enable students to move across pathways as their interests and aspirations evolve; and
- **individualized academic, career, and other student supports** that respect and promote student and family decision making and that enable students to explore options, make strategic choices, and set and achieve informed goals.

To support the first aim of the Launch Years initiative—to improve learning opportunities for each student during the last two years of high school and into the transition to postsecondary education—the Charles A. Dana Center at The University of Texas at Austin has collaborated with stakeholders from K–12 and higher education to develop a course framework for a senior-level transition mathematics course.

A design team, comprising content experts in K–12 and higher education, worked together to develop the course framework and sought input from experts in a variety of fields to inform the design team's work. The framework contained in this document describes a course that encompasses multiple pathways and also supports students' social, emotional, and academic development—an often-overlooked aspect of education that research indicates is crucial to students' ability to thrive in school, career, and life.

We recognize that simply implementing new courses is not enough. We commit to challenging and eliminating institutional and systemic barriers to students' opportunities to access—and succeed in—mathematics. This commitment includes proactively working with partners to change institutional cultures and educator mindsets toward recognizing and building upon student assets and student strengths.

Acknowledgments

Design Team

Ted Coe
Director of Content Advocacy and Design, NWEA (Oregon)

Jacqueline Coomes
Professor, Department of Mathematics, Eastern Washington University

Arlene Crum
Director of Mathematics, Learning and Teaching, Office of the Superintendent of Public Instruction, Washington

Mary F. Davis
Professional Learning Facilitator, Mathematics, K12, Charles A. Dana Center (Texas)

Rebecca Gammill
Mathematics Teacher, Kennesaw Mountain Academy of Mathematics, Science, and Technology (Georgia)

Brent Griffin
Director of Retention & Graduation, Georgia Institute of Technology

Tony Q. Martinez
Secondary Mathematics Coordinator, Leander Independent School District (Texas)

Josh Recio
Course Program Specialist, Secondary Mathematics, K12, Charles. A. Dana Center (Texas)

Frank Savina
Course Program Specialist, Mathematics, Higher Education, Charles A. Dana Center (Texas)

Prudence York–Hammons
Mathematics Faculty Temple College (Texas)

Other Contributors

Robert Berry, III
Samuel Braley Gray Professor of Mathematics Education, University of Virginia

Carla Bidwell
Secondary Mathematics Specialist, Metropolitan Regional Educational Service Agency (Georgia)

Kathi Cook (Ret.)
Manager, K12 Course Programs, Charles A. Dana Center (Texas)

Mark Daniels
Clinical Professor of Mathematics, University of Texas at Austin

Phil Daro
Director of Mathematics, Strategic Education Research Partnership (SERP) Institute (California)

Tristan Denley
Executive Vice Chancellor for Academic Affairs and Chief Academic Officer, University System of Georgia

Roshele Friudenberg
Chair, Developmental Education Division, Lee College (Texas)

Coretta Hoffman
Assistant Principal, Mead High School (Washington)

Nicholas Horton
Beitzel Professor in Technology and Society (Statistics and Data Science), Amherst College (Massachusetts)

Chris S. Hulleman
Associate Professor, Center for Advanced Study of Teaching and Learning, Curry School of Education, University of Virginia

Bob Klein
Past President, Benjamin Banneker Association, Inc. (Arkansas)

Brooke Kline (Ret.)
Mathematics Secondary Program Specialist, Teaching and Learning Team, Georgia Department of Education

Rachel Levy
Deputy Executive Director, Mathematical Association of America (Washington, DC)

Susan May
Course Program Specialist, Secondary Mathematics, K12, Charles. A. Dana Center (Texas)

Bill Moore
Director for K-12 Partnerships, Washington State Board for Community and Technical Colleges

Debbie Olson
CEO, Mathematics Education Collaborative (Washington)

Roxy Peck
Professor Emerita of Statistics, Cal Poly, San Luis Obispo (California)

Brea Ratliff
Past President, Benjamin Banneker Association, Inc. (Arkansas)

Connie Richardson
Manager, Higher Education Course Programs, Charles A. Dana Center (Texas)

Shane Rosenkrantz
Technology Design Coach, Austin Independent School District (Texas)

Michael Steele
Professor of Mathematics Education, University of Wisconsin–Milwaukee

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Introduction and Purpose

This framework is designed to guide in the development of courses that will prepare students for the relevant mathematics needed for success in postsecondary education and future careers. It provides the opportunity for students to demonstrate proficiency in working with quantities, functions, and probability and statistics, appropriate to a third-year high school credit in mathematics. The course is intended to meet the needs of **all** students who are prepared to enroll in a traditional Algebra II course, not only those on the path to calculus. Upon successful completion, students will be ready to enter follow-on courses in quantitative literacy, introductory data science, AP/IB Statistics, or precalculus.

A course based on this framework requires supporting students' social, emotional, and academic development (SEAD) through explicit instruction about and development of skills and strategies needed to succeed after high school. Courses implemented from this framework should also explicitly attend to the systemic inequities in America's high schools through placement, support, and pedagogical practices. Finally, the framework requires that the courses develop students' ability to engage in important mathematical practices and processes, including:

- Working with quantitative information and mathematical and statistical concepts that use language and representations to make decisions, solving problems, formulating functions that model real-world situations, and drawing inferences from data analysis;
- Making sense of problems and developing strategies to find solutions and persevere in solving them;
- Reasoning, modeling, and making decisions with given information, including understanding and critiquing the arguments of others; and
- Selecting and using technology appropriate to a given context.

Course Design Principles

Curricular materials and classroom instruction for this course should engage students in meaningful interactions that amplify the learning through social interaction; facilitate transfer of math and SEAD skills; and create an inclusive learning context for all learners, particularly for students who feel disconnected from mathematics and disaffected by the learning process.

The following design principles describe how curricular materials and classroom instruction for the transition course should be structured to support a coherent and engaging experience. Developers should use these standards to create curricular materials that are true to the vision of the course, and educators should also use the design principles when building a repertoire of pedagogical strategies for use in teaching the course.

We are aware that many students and teachers already engage in these behaviors. Our hope is that these design principles will be seen as reinforcing and supportive. The spirit of this framework recognizes that, at some levels, we are all learners, and are growing in our understanding of mathematics, one another, and the world around us.

Modern Algebra II Course Design Principles

Design Principle	Students will . . .	Teachers will . . .
Active Learning. The course provides regular opportunities for students to actively engage in discussions and tasks using a variety of different instructional strategies (e.g., hands-on and technology-based activities, small group collaborative work, facilitated student discourse, interactive lectures).	• Be active and engaged participants in discussion, in working on tasks with classmates, and in making decisions about the direction of instruction based on their work. • Actively support one another's learning. • Discuss course assignments and concepts with the instructor and/or classmates outside of class.	• Provide activities and tasks with accessible entry points that present meaningful opportunities for student exploration and co-creation of mathematical understanding. • Facilitate students' active learning of mathematics and statistics through a variety of instructional strategies, including inquiry, problem solving, critical thinking, and reflection, with limited time spent in "direct teach" activities. • Create a safe, student-driven classroom environment in which all students feel a sense of belonging to the class and the discipline, are not afraid to take risks or make mistakes, and are able to make decisions about the direction for instruction through the results of their exploration of mathematics and statistics.

Modern Algebra II Course Design Principles

Design Principle	Students will . . .	Teachers will . . .
Constructive Perseverance. The course supports students in developing the tenacity, persistence, and perseverance necessary for learning mathematics and statistics, for using mathematics and statistics to tackle authentic problems, and for being successful in post-high school endeavors.	• Make sense of tasks by drawing on and making connections with their prior understanding and ideas. • Persevere in solving problems and realize that it is acceptable to say, "I don't know how to proceed here," but that it is not acceptable to give up; seek help from appropriate sources to continue to move forward. • Compassionately help one another by sharing strategies and solution paths rather than simply giving answers. • Reflect on mistakes and misconceptions to improve their mathematical understanding. • Seek to understand and address the reasons for their struggles to help them make progress in solving problems and overcoming challenges in the course. • Understand that, while they may struggle at times with mathematics tasks, breakthroughs often emerge from confusion and struggle.	• Provide instruction and information about the role of productive struggle in learning. • Pose tasks on a regular basis that require a high level of cognitive demand. • Allow students to engage in productive struggle with challenging tasks. • Anticipate what students might struggle with during a lesson and be prepared to support them productively through the struggle. • Give students time to struggle with tasks and ask questions that scaffold students' thinking without stepping in to do the work for them. • Praise students for their effective efforts in making sense of mathematical ideas and for their perseverance in reasoning through problems and in overcoming setbacks and challenges in the course. • Help students realize that confusion and errors are a natural part of learning by facilitating discussions on mistakes, misconceptions, and struggles. • Provide students with low-stakes opportunities to fail and learn from failure. • Provide regular opportunities for students to self-monitor, evaluate, and reflect on their learning, both individually and with their peers.

Modern Algebra II Course Design Principles

Design Principle	Students will . . .	Teachers will . . .
Problem Solving. The course provides opportunities for students to make sense of problems and persist in solving them.	• Apply previously learned strategies to solve unfamiliar problems. • Explore and use multiple solution methods. • Share and discuss different solution methods. • Be willing to make and learn from mistakes in the problem-solving process. • Use tools and representations, as needed, to support their thinking and problem solving.	• Present tasks that require students to find or develop a solution method. • Provide tasks that allow for multiple strategies and solution methods, including transfer of previously developed skills and strategies to new contexts. • Provide opportunities to share and discuss different solution methods. • Model the problem-solving process using various strategies. • Encourage and support students to explore and use a variety of approaches and strategies to make sense of and solve problems.
Authenticity. The course presents mathematics and statistics as necessary tools to model and solve problems that arise in the real world.	• Recognize specific ways in which mathematics is used in everyday decision making. • Recognize problems that arise in the real world that can be solved with mathematics or statistics. • Contribute meaningful questions that can be answered using mathematics.	• Provide opportunities to solve problems that are relevant to students, both in class and on assessments, that utilize real-world—not contrived—contexts. • Provide opportunities for students to pose questions that can be answered using mathematics or statistics and answer them.
Context and Interdisciplinary Connections. The course presents mathematics and statistics in context and connects mathematics and statistics to various disciplines and everyday experiences.	• Contribute personal experiences, where appropriate, that connect to classroom experiences. • Actively seek connections between classroom experiences and the world outside of class.	• Provide opportunities for students to share their personal backgrounds and interests, including cultural values, and help make the connection between what is important in students' lives and future aspirations, and what they are learning in mathematics. • Provide activities and tasks that use real data, whenever possible. • Provide activities and tasks that illustrate authentic applications. • Provide activities and tasks that explore problems from a variety of academic disciplines, programs of study, and careers, and that are culturally relevant.

Modern Algebra II Course Design Principles

Design Principle	Students will . . .	Teachers will . . .
Communication. The course develops students' ability to communicate about and with mathematics and statistics in contextual situations.	• Present and explain ideas, reasoning, and representations to one another in pair, small-group, and whole-class discourse using discipline-specific terminology, language constructs, and symbols. • Seek to understand the approaches used by peers by asking clarifying questions, trying out others' strategies, and describing the approaches used by others. • Listen carefully to and critique the reasoning of peers using examples to support or counterexamples to refute arguments. • Justify mathematical reasoning with clarity and precision.	• Introduce concepts in a way that connects students' experiences to course content and that bridges from informal contextual descriptions to formal definitions. • Clarify the use of mathematical and statistical terminology and symbols, especially those used in different contexts or different disciplines. • Engage students in purposeful sharing of mathematical ideas, reasoning, and approaches using varied representations. • Support students in developing active listening skills and in asking clarifying questions to their peers in a respectful manner that deepen understanding. • Facilitate discourse by positioning students as authors of ideas who explain and defend their approaches. • Provide regular opportunities for students to write about mathematics and statistics with tasks to deepen understanding and with authentic contextual tasks that require use of mathematical or statistical concepts (e.g., writing a brief paper that interprets the results of a statistical study). • Scaffold instruction to support students in developing the required reading and writing skills.

Modern Algebra II Course Design Principles

Design Principle	Students will . . .	Teachers will . . .
Technology. The course leverages technology to develop conceptual understanding and to facilitate active learning by enabling students to directly engage with and use mathematical concepts.	• Use technology to assist them in visualizing and understanding important mathematical concepts and as a support to problem solving. • Allow technology to assist investigations with problems that might otherwise be too difficult or time-consuming to explore. • Consider the relative usefulness of a range of tools in particular contexts. • Understand that the use of tools or technology does not replace the need for an understanding of reasonableness of results or how the results apply to a given context.	• Use technology to assist students in visualizing and understanding important mathematical concepts and support students' mathematical reasoning and problem solving. • Leverage technology as a tool that can expand the scope of mathematical ideas and problems that students can investigate. • Support students in using technology for more than just answer-getting and in making appropriate choices of technology to use, depending on the problem to be solved. • Be mindful of effective uses of technology and plan carefully for strategic use of technology.

Outcomes for Integrating Social, Emotional, and Academic Development

In order for students to acquire the knowledge and skills outlined in the Integrating SEAD objectives below, materials for the Modern Algebra II course should provide explicit instruction for each outcome, combined with opportunities for students to apply what they have learned as they engage with the mathematics content. The course should strengthen social–emotional skills and competencies critical to academic and life success, including competencies in the cognitive, social and interpersonal, and emotional domains.

To be proficient in this course, students should:

Use collaboration and communication as a means for learning. Recognize situations for which collaboration is an effective strategy, identify the features of collaborative work groups, and develop strategies for overcoming group work challenges. Work collaboratively with students from various cultural and ethnic backgrounds while examining alternate points of view. Accept constructive criticism and revise personal views when evidence warrants.

Utilize resources to overcome obstacles. Engage in productive academic behaviors, including recognizing when help is needed with a task, and developing and applying a variety of strategies and sources for seeking help; monitoring and adjusting attitudes, emotions, and thoughts when facing challenging tasks or academic setbacks; and seeking and using feedback to improve performance.

Recognize and improve individual behaviors. Maintain motivation and persistence through a variety of strategies, including identifying and adjusting habits and beliefs that have interfered with success; applying metacognitive awareness to plan, monitor, evaluate, and reflect on their learning; and setting and monitoring goals.

Content Outcomes

To acquire the knowledge and skills outlined in the content outcomes, student learning should be motivated by an authentic context, and they apply their knowledge and skills to solve real-world problems appropriate for and of interest to students in high school.

Content outcomes are presented here as overarching mathematical concepts with supporting examples.

Modeling with Quantities

Work with quantities to make decisions, solve problems, and communicate solutions. Students need to demonstrate an understanding of the real number system, using technology such as calculators, spreadsheets, and programming languages to perform calculations. They should be able to demonstrate relationships between numbers and interpret quantities in context. Because units are crucial to real-world situations, students should use units when presenting solutions. They should provide justification when estimating or rounding numbers, choosing a level of accuracy appropriate for reporting quantitates based on limitations in measurement. They should use previously learned skills in solving problems that are grade-level appropriate. Examples of proficiency include the ability to perform dimensional analysis in context; convert units and rates; and determine, identify, and use quantities to model a given situation, explaining why the model is appropriate.

Modeling with Data

Acknowledge and describe variability, and work with variability in data. In order to describe variability, students need to be able to understand possible sources of variability. According to the GAISE Pre-K–12 report, "statistical problem solving and decision making depend on understanding, explaining, and quantifying the variability in the data withing the given context" (p. 7).[1] Students need to analyze data numerically and graphically to describe variability in data. Examples of proficiency include the ability to analyze a study with attention to error, use technology to calculate quantities of variability, including IQR and standard deviation, demonstrate an understanding of these quantities, and describe and model variability using distributions.

Analyze and summarize univariate data. Summarizing univariate data lays the foundation for multivariate thinking. Students should be able to communicate analyses of univariate data and compare multiple distributions of data in order to solve problems. Examples of proficiency include the ability to describe distributions with the same mean but different standard deviation, use the Empirical Rule to answer questions in context, and interpret and analyze key attributes of the graph of a distribution, using these attributes to compare multiple data distributions.

Analyze and summarize multivariate data. Many statistical analyses involve more than one variable. Students should analyze data involving two or more quantitative variables, using technology such as spreadsheets, statistical packages, or calculators. They should be able to use regression to solve problems and make decisions as part of the modeling process, and make their reasoning clear. Examples of proficiency include the ability to use technology to create scatterplots and analyze for patterns, linearity, outliers, and influential points, explaining the effect of individual data points to a data set, and interpret regression models.

Calculate, estimate, and interpret probabilities. Using probability to make decisions should connect to real-life events, and in doing so, students should be able to communicate the reasoning behind their decisions. Students should be conducting simulations to model real-world phenomena and estimate probabilities. Tools such as tree diagrams, Venn diagrams, and two-way tables should be utilized to organize information when determining probabilities. Opportunities to interpret real-life problems can be drawn from epidemiological data representations, and from podcasts and their related websites such as Freakonomics and FiveThirtyEight. Examples of proficiency include the ability to determine the number of ways an event may occur, determine the tool most appropriate for representing outcomes, and explain the meaning of conditional probability in contextual situations.

Summarize statistical information. Communicating quantitative information using statistical language in oral, written, and graphical form is key to building data acumen. After analyzing data using graphical and numerical methods, students need to show proficiency in communicating their conclusions, including summarizing results from statistical studies using appropriate statistical justifications. Examples of proficiency include the ability to use appropriate symbols, distinguishing between statistics and

1 American Statistical Association. (2020). *Pre-K-12 Guidelines for Assessment and Instruction in Statistics Education II (GAISE II): A Framework for Statistics and Data Science Eduction.* Retrieved from https://www.amstat.org/asa/files/pdfs/GAISE/GAISEIIPreK-12_Full.pdf

parameters, explain which type of graphical representation is best for summarizing data, and describe when and why the median would be a better measure of center than the mean.

Modeling with Functions

Understand and apply the foundations of functions. An understanding of functions that are grounded in quantities is fundamental. Since all functions in the course should be applicable to an authentic application, students should be able to interpret units of input and resulting units of output in context, thereby recognizing attributes such as domain and range. To demonstrate this understanding, students should compare functions in different forms and use multiple representations, such as formulas, equations, graphs, and tables to highlight and investigate quantities using appropriate function notation. Students should see that members of the same function family have distinguishing attributes common to all functions within that family. The use of graphing technology, including online graphing calculators, should be routine for students when working with and analyzing functions. Examples of proficiency include the ability to explain which representation of a function is most appropriate in a given context, evaluate a function for a given value, and use various strategies to find input values for different output values.

Understand and apply functions as relations whose quantities change in tandem. In order to develop function reasoning, students must view a function as a relationship of two quantities that vary in tandem. That is, a function describes how a change in one quantity corresponds to a change in another. This dynamic, covariational interpretation of functions is essential, and students should frequently describe this relationship. Examples of proficiency include the ability to distinguish between dependent and independent variables, recognize and describe key features of functions and their meanings based on changes to the variables, interpret and communicate the behavior of functions, and identify constraints on domains.

Synthesize and apply function understandings to model the real world. The study of quantities, algebra, and functions is of most use when it is connected to describing real-world phenomena. Students must engage regularly in the modeling process to describe the world in which they live. This modeling process entails the critical skills of making and changing assumptions, assigning variables, determining whether linear, quadratic, simple rational or exponential relationships may be appropriate and finding solutions to contextual problems. The ability to read, recognize, and solve problems involving linear, quadratic, simple rational, and exponential relationships may be appropriate, but the analysis should be based on how the quantities relate to each other, not simply on the shape of the data when graphed. Students will therefore need to understand how quantities relate to justify the use of a model. Examples of proficiency include the ability to model percent change, select an appropriate function that appears to model a relationship, describe a situation based on its function type, and develop a function from a meaningful problem.

Use functions to create new functions. Students should recognize when and how to operate on functions. The need to apply operations may involve a context where it makes sense to add or subtract functions, combine functions, “shift” or “stretch” functions, or invert functions to serve some other need. Such transformations should be accompanied by explanations of the results in terms of the quantities, and not simply in terms of shifting shapes. Examples of proficiency include the ability to use parent functions as the basis for transformations to graph new functions, identifying and interpreting key features of the new functions; and construct, describe, and use the inverse of a linear, quadratic, or exponential function.

Understand algebraic structures and extend algebraic skills. Technology can manipulate algebraic expressions and solve problems with incredible speed and accuracy. Technology should be leveraged for problem solving. However, students should also have a baseline set of understandings and algebraic and graphing skills to confirm an output from technology. Students' ability to explain why what they do in solving different types of equations works, and identify where shortcomings may arise, validates this understanding. Examples of proficiency include the ability to solve authentic linear, quadratic, and exponential equations including systems of linear equations and systems of equations, explain the meaning of a solution to an equation, assess the reasonableness of solutions, and rewrite formulas for a specified variable.

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